NPG-184

NPS55-81-021

NAVAL POSTGRADUATE SCHOOL
Monterey, California

THESIS

Investigation of Alternative Methods
Including Jackknifing for Estimating
Point Availability of a System

Barbaros Aba

September 1981

Thesis Advisor: D. P. Gaver

Approved for public release; distribution unlimited

Prepared for:
Naval Postgraduate School
Monterey, California

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral J.J. Ekelund
Superintendent

David A. Schrady
Provost

This thesis prepared in conjunction with research supported in part by Naval Postgraduate School.

Released as a
Technical Report by:

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER NPS55-81-021	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Investigation of Alternative Methods Including Jackknifing for Estimating Point Availability of a System		5. TYPE OF REPORT & PERIOD COVERED Thesis Report Nov 1980 - Sep 1981
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Barbaros Aba		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93940		12. REPORT DATE September 1981
		13. NUMBER OF PAGES 80
14. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Jackknife Availability
Likelihood
Log-normal

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Properties of two alternative procedures to the Jackknife Point and Confidence Interval Estimation Procedure of Gaver and Chu have been studied. They are called the Log-Normal Likelihood Procedure (LNLJ) and the Moment Procedure (MP). These two procedures were investigated and compared with the Jackknife Point and Confidence Interval Availability Estimation Procedure. Numerical results from simulations are presented in this report.

Approved for public release, distribution unlimited.

Investigation of Alternative Methods Including Jackknifing for Estimating Point Availability of a System

by

Barbaros Aba
Lieutenant, Turkish Navy
B.S., U.S. Naval Postgraduate School, 1981

Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September, 1981

ABSTRACT

Properties of two alternative procedures to the Jackknife Point and Confidence Interval Estimation Procedure of Gaver and Chu have been studied. They are called the Log-Normal Likelihood Procedure (LNLJ) and the Moment Procedure (MP). These two procedures were investigated and compared with the Jackknife Point and Confidence Interval Availability Estimation Procedure. Numerical results from simulations are presented in this report.

TABLE OF CONTENTS

LIST OF TABLES

ACKNOWLEDGMENT

The author wishes to express his appreciation to the individuals who have given their time in this effort. Specifically, I would like to recognize Professor D. P. Gaver for his guidence, assistance and patience; Professor J. D. Esary, who guided me while Professor D. P. Gaver was out of the U.S.A; Professor R. N. Forrest as the second reader; and my wife, Fügen, for her patience during this study.

I. INTRODUCTION

A. OVERVIEW

Availability is the measure of equipment effectiveness that relates reliability and maintainability to operational readiness. In some cases, availability and operational readiness have been considered the same. In general, these are all requirements which must be considered during the design and operation of a system, and, thus, must be quantitatively evaluated.

There are a variety of ways of expressing availability. In general, availability relates up time (reliability-related) to down time (maintainability- related), and it may be defined as the ratio between the time the system is capable of performing its mission to the total time the system is in operational demand. Alternatively, it is a measure of the probability that a single system is "up" or available at a possibly "random" point in time when its services are needed.

The three expressions of availability of greatest concern are -- (1) inherent availability (Ai), (2) operational

availability (Ao), and (3) achieved availability (Aa). See Rise and Bjorklund[Ref. 1]. In this study, we will be concerned with only the inherent availability. Often, inherent availability, which is a hardware oriented measure, is specified and required within a maintainability contract requirement.

A matter of primary concern is that of estimating availability from observations on system up times and down times. The estimating process should furnish both a point, or single number, estimate and also some measure of the stability of the estimate such as a standard error, or confidence limits. The latter problem is more difficult than the former.

In the paper by Gaver and Chu[Ref. 2], it has been demonstrated that the jackknife technique can be useful for confidence interval estimation of system availability. For instance, let

$$A = \frac{E(U)}{E(U)+E(D)} \qquad (1.1)$$

be the long run system availability of a single unit system that changes from up to down states in accordance with a two-state semi-Markov (or more general) process for which up [down] state expected duration is $E(U)[E(D)]$. Then it is

possible to successfully jackknife the naive point estimator that involves replacing expectations by the corresponding sample means,

$$\tilde{A} = \frac{\bar{U}}{\bar{U} + \bar{D}} \qquad (1.2)$$

after initial logistic transformation. There are, however, particular parametric families of distributions, popular as models for summarizing up and down time data, for which the most statistically efficient estimators of E(U) and E(D) are not the simple sample arithmetic means.In particular, this is true for,

(1).The log-normal distribution with unknown (to be estimated) parameters; often this is used as a model for down or repair times. In particular, the log normal distribution has enjoyed considerable popularity for representing electronic system repair times.See Kline and Almog[Ref. 3].

(2).the general gamma distribution with unknown shape and scale parameters, and

(3).the general Weibull distribution with unknown shape and scale parameters.

B. PURPOSE AND APPROACH

1. Objectives

Availability estimation by use of the jackknife has already been studied by Gaver and Chu.This thesis presents two procedures alternative to the above. These may be called the Log-normal Likelihood jackknife (LNLJ) procedure and the Moments procedure (MP). In brief, the jackknife method has the capacity to reduce the statistical bias of estimates of such quantities as system availability, and also, and more importantly, to furnish confidence limits that behave in a satisfactory manner (economically enclose the true availability) despite the fact that underlying distributions are unknown. See Miller[Ref. 4] for a review of much of the literature on jackknifing.

The present estimation procedure makes use of the specific assumption of a log normal model for repair times to compute the maximum likelihood estimate (mle) of availability. Properties of the procedure have been studied by Monte Carlo simulation. A number of such simulation results are presented in this thesis. Also presented are comparisons with the direct method of Gaver and Chu, and with a computationally simple method based on moment estimation.

Attention also has been paid to study of the sensitivity of the LNLJ procedure to specification error. This refers to the error introduced by assumption of a log-normal distributional model and the corresponding maximum likelihood estimates when, in fact, another distribution governs the observations. It is found that LNLJ method, which is theoretically most efficient in large samples if the model (likelihood) is correct, can sometimes produce consistently biased results when the log normal model does not apply. Since it is nearly impossible to assure log normality from small samples of data, this finding suggests that caution is in order.

2. Log normal Likelihood Procedure For a Single Unit

(a). Data is given as follows,

$u_1, u_2, \ldots\ldots\ldots\ldots, u_n$ (up times)

$d_1, d_2, \ldots\ldots\ldots\ldots, d_n$ (down times)

$x_1 = \ln d_1, x_2 = \ln d_2, \ldots\ldots\ldots, x_n = \ln d_n$

(b). Assume down times are log-normal. Then,

$$E[D] = \exp(\mu_d + 1/2 \cdot \sigma_d^2) \quad (1.2.1)$$

where

$$\mu_d = E[\ln D] \quad (1.2.2)$$

$$\sigma_d^2 = Var[\ln D] \quad (1.2.3)$$

Furthermore, the maximum likelihood estimates (mle) of μ_d and σ_d^2 are

$$\hat{\mu}_d = \bar{x} = 1/n \, \Sigma \ln(d_i) \tag{1.2.4}$$

$$\hat{\sigma}_d^2 = 1/n \, \Sigma \, (x_i - \bar{x})^2 \, , \tag{1.2.5}$$

so the mle of E[D] is, by invariance,

$$\widetilde{E}[D] = \exp(\breve{\mu}_d + 1/2 \bullet \breve{\sigma}_d^2) \tag{1.2.6}$$

it is essentially the latter expression that will be used in the estimate for availability to replace the simple first moment estimate $E(D)=\bar{d}$. In fact, $\tilde{\sigma}_d^2$ has been replaced by its unbiased version (n replaced by n-1).

(c). If the above holds, it is advantageous to transform first (See Mosteller and Tukey[Ref. 5]) the estimated availability

$$A = \frac{E(U)}{E(U)+E(D)} \quad ;$$

the log-logistic transform of availability is

$$\ln(\tilde{A} \, / \, 1-\tilde{A}) = \ln \widetilde{E}(u) - (\breve{\mu}_d + 1/2 \bullet \tilde{\sigma}_d^2) \tag{1.2.7}$$

Jackknifing will be carried out using the statistic

$$L_{LN} = \ln(\tilde{A} \, / \, 1-\tilde{A}) = \ln(\bar{u}) - (\bar{x} + 1/2 \bullet s_x^2) \tag{1.2.8}$$

(d). Recompute L_{LN} repeatedly, leaving out successively the sample pairs (u_1,d_1), (u_2,d_2), ..., (u_n,d_n)

$$\bar{u}_{-j} = 1/n-1 \bullet \{ \sum_{i=1}^{j-1} u_i + \sum_{i=j+1}^{n} u_i \} \qquad (1.2.9)$$

$$x_{-j} = 1/n-1 \bullet \{ \sum_{i=1}^{j-1} x_i + \sum_{i=j+1}^{n} x_i \} \qquad (1.2.10)$$

$$s_{x-j}^2 = 1/n-2 \bullet \{ \sum_{i=1}^{j-1} (x_i - \bar{x}_{-j})^2 + \sum_{i=j+1}^{n} (x_i - \bar{x}_{-j})^2 \} \qquad (1.1.11)$$

$$L_{LN,j} = \ln(\bar{u}) - (\bar{x}_j + 1/2 \bullet s_{x,j}^2) \qquad (1.2.12)$$

where $j=1,2,.......,n$

(e). Compute the pseudovalues as follows:

$$P_{LN,j} = nL_{LN} - (n-1) L_{LN,-j} \quad j=1,2,....,n \qquad (1.2.13)$$

Recall that $L_{LN} = L_{LN,all}$ is the result of the computing the quantity to be jackknifed, leaving out none of data.

(f). Compute the mean and variance of the pseudovalues,

$$\bar{P}_{LN} = 1/n \bullet \sum_{j=1}^{n} P_{LN,j} \qquad (1.2.14)$$

$$S_{LN}^2 = 1/n-1 \bullet \sum_{j=1}^{n} (P_{LN,j} - \bar{P}_{LN})^2 \qquad (1.2.15)$$

(g). The jackknifed point estimate of the availability is now,

$$A = \exp(\bar{P}_{LN}) \; / \; 1+\exp(\bar{P}_{LN}) \qquad (1.2.16)$$

(h). Symmetric two sided confidence limits at confidence level $(1-\alpha)$% are derived as follows;

$$CL_{LN} = \bar{P}_{LN} - t_{1-\alpha/2}(n-1) \; SQRT(1/n \bullet S_{LN}^2) \qquad (1.2.17)$$

$$CU_{LN} = \bar{P}_{LN} + t_{1-\alpha/2}(n-1)\ \mathrm{SQRT}(1/n \bullet S_{LN}^2) \qquad (1.2.18)$$

where $t_{1-\alpha/2}(n-1)$ is the $(1-\alpha/2)100\%$ quantile of Student's-t with n-1 degrees of freedom. Then,

$$\frac{\exp(CL_{LN})}{1+\exp(CL_{LN})} \le A \le \frac{\exp(CU_{LN})}{1+\exp(CU_{LN})} \qquad (1.2.19)$$

with confidence approximately $(1-\alpha)100\%$. Note that the confidence limits are nearly symmetric around ln(E[U]/E[D]), and not around A.

(i).One sided confidence limits at confidence level $(1-\alpha)100\%$ are derived

$$CL_{LN} = \bar{P}_{LN} - t_{1-\alpha}(n-1)\ \mathrm{SQRT}(1/n \bullet S_{LN}^2) \qquad (1.2.20)$$

$$CU_{LN} = \bar{P}_{LN} + t_{1-\alpha}(n-1)\ \mathrm{SQRT}(1/n \bullet S_{LN}^2) \qquad (1.2.21)$$

So, one-sided upper confidence limit is

$$A \le \frac{\exp(CU_{LN})}{1+\exp(CU_{LN})} \qquad (1.2.22)$$

and lower confidence limit is

$$A \le \frac{\exp(CL_{LN})}{1+\exp(CL_{LN})} \qquad (1.2.23)$$

Note that if up times are exponentially distributed then u is actually the mle of E[U], so under the model assumptions the (transformed) mle of availability is being

jackknifed; this procedure has been validated by Reeds [Ref. 6] for large samples. Simulation studies are nevertheless essential for investigating properties of the jackknife or alternative techniques for small samples, and for investigating sensitivity to model assumption.

3. <u>Moment Procedure For a Single Unit</u>

(a). Data given as follows

$u_1, u_2, \ldots\ldots, u_n$ (up times)

$d_1, d_2, \ldots\ldots, d_n$ (down times)

$y_1 = \ln u_1, y_2 = \ln u_2, \ldots\ldots, y_n = \ln u_n$

$x_1 = \ln d_1, x_2 = \ln d_2, \ldots\ldots, x_n = \ln d_n$

(b). Assume up and down times are log-normal. Then,

$$E[U] = \exp(\mu_u + 1/2 \cdot \sigma_u^2) \quad (1.3.1)$$

$$E[D] = \exp(\mu_d + 1/2 \cdot \sigma_d^2) \quad (1.3.2)$$

where,

$$\mu_u = E[\ln U] \quad (1.3.3)$$

$$\mu_d = E[\ln D] \quad (1.3.4)$$

$$\sigma_u^2 = Var[\ln U] \quad (1.3.5)$$

$$\sigma_d^2 = Var[\ln D] \quad (1.3.6)$$

Furthermore, the mle of μ_u, σ_u^2, μ_d, and σ_d^2 are

$$\tilde{\mu}_u = \bar{y} = 1/n \cdot \sum_{i=1}^{n} y_i \quad (1.3.7)$$

$$\tilde{\sigma}_u^2 = 1/n \cdot \sum_{i=1}^{n} (y_i - \bar{y})^2 \quad (1.3.8)$$

$$\tilde{\mu}_d = \bar{x} = 1/n \cdot \sum_{i=1}^{n} x_i \quad (1.3.9)$$

$$\tilde{\sigma}_d^2 = 1/n \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2 \qquad (1.3.10)$$

so the mle of E[U] and E[D] are, by invariance,

$$E[U] = \exp(\tilde{\mu}_u + 1/2 \cdot \tilde{\sigma}_u^2) \qquad (1.3.11)$$

$$E[D] = \exp(\tilde{\mu}_d + 1/2 \cdot \tilde{\sigma}_d^2) \qquad (1.3.12)$$

(c) If above holds, the logistic transform of availability is,

$$\ln (\tilde{A} / (1-\tilde{A})) = (\tilde{\mu}_u + 1/2\, \tilde{\sigma}_u^2) - (\tilde{\mu}_d + 1/2\, \tilde{\sigma}_d^2) \qquad (1.3.13)$$

estimation will be carried out using the statistic

$$P = \widehat{\ln} (A / (1-A)) = (\bar{y} + 1/2 \cdot s_y^2) - (\bar{x} + 1/2 \cdot s_x^2) \qquad (1.3.14)$$

(d) Simply the point estimate of availability is now,

$$A = \exp(P) / 1 + \exp(P) \qquad (1.3.15)$$

(e) Variance of the statistic P can be calculated as follows,

$$\mathrm{Var}(P) = (SE)^2 = 1/n \cdot s_y^2 + 1/4 \cdot \mathrm{Var}[s_y^2) + 1/n \cdot s_x^2 + 1/4 \cdot \mathrm{Var}[s_x^2] \qquad (1.3.16)$$

where,

$$\mathrm{Var}[s_y^2] = 1/n \; [M_{4y} - (s_y^2)^2] \qquad (1.3.17)$$

$$\mathrm{Var}[s_x^2] = 1/n \; [M_{4x} - (s_x^2)^2] \qquad (1.3.18)$$

(f) Symmetric two sided confidence limits at confidence level $(1-\alpha)\%$ are derived as follows,

$$L = P - z_{1-\alpha/2} (SE)^2 \qquad (1.3.19)$$

$$U = p + z_{1-\alpha/2}(SE)^2 \qquad (1.3.20)$$

where $z_{1-\alpha/2}$ is the $(1-\alpha/2)100\%$ quantile of the standard normal distribution. Then,

$$\{\exp(L) / 1+\exp(L)\} \leq A \leq \{\exp(U) / 1+\exp(U)\} \qquad (1.3.21)$$

II. SIMULATION PROCEDURE

A simulation procedure has been used to compare the operating characteristics of LNLJ methodology introduced here with other approaches. Specifically, simulation has been used to compute

(a) the actual coverage of the true availability figure, A, by the confidence intervals given by the procedures under study, when the nominal coverage is $(1-\alpha)100\%$,

(b) measures of confidence interval expected width and variance width,

(c) estimate of the expected point availability estimated by the procedure under study.

The simulation program was written in FORTRAN IV, and the simulation have been carried out on the IBM 3033 at the Naval Postgraduate School. A more detailed description of the program and its major subroutines is given in Appendix A.

An outline of the simulation procedure now follows;

(A) A simulated sample of up-time durations, $(u_i, i=1, 2,\ldots, n)$ and down-time durations, $(d_i, i=1, 2,\ldots,n)$ were

times. The Naval Postgraduate School LLRANDOM package was used, along with the International Mathematical and Statistical Library (IMSL) random number generator. For all cases considered (exceptions noted), expected up and down times were $E[U]=73.036$ and $E[D]=3.844231$ for the population sampled, so long-run availability $A=0.95$. Sample sizes of $n=15$ and $n=25$ were simulated, and provided the bases for point estimators and for confidence intervals.

(B) Here are the different distributional situations that were sampled. A total of 1000 replications was used to evaluate each procedure in each distributional situation.

(1) Up-times come independently from the iid Expon(λ) distributions. Down-times come independently from the log-normal(μ,σ^2) distribution.

(2) Up-times are iid Expon(λ). Down times are iid Expon(μ).

(3) Up-times are iid Expon(λ). Down-times are iid Gamma(θ,k), being a scale and k a shape parameter, so arranged that $E[D]=\mu^{-1}$ and $Var[D]=(\sqrt{k}\mu)^{-2}$. A Gamma with $k>1$, e.g. $k=2$, represents data that is more tightly grouped around its mean than is true of exponentially distributed data; it roughly resembles

data that is log-normal. A gamma with $k<1$, e.g. $k=1/2$, represents data of relatively extreme dispersion and positive skewness as compared to exponential data having a very long right tail, compensated for by a high density near zero. A gamma distribution with integer k ($k=1,2,3,\ldots.$) is often called Erlang; realizations are easily simulated by summing k independent Exponential realizations.

(4) Up-times are iid Expon(λ). Down-times are iid long-tailed h distribution:

$$d=E[D]\bullet w\bullet \exp(h\bullet w)\bullet(1-h)^2, \quad 0<h<1,$$

where w comes from Expon(1). This long-tailed h distribution possesses no closed-form representation. However, it can be shown to have characteristics similar to those of the Exponential for small values of h, but to have a systematically longer right tail than the exponential.

(5) Up times are iid long-tailed h distribution. Down times are iid Expon(μ).

(6) Up times are iid Expon(λ). Down times are iid Weibull(θ,k), being a scale parameter and k a shape parameter, so arranged that $E[D]=\theta^{1/k}\bullet\Gamma(1/k + 1)$ and

$Var[D] = \theta^{2/k} \cdot [\Gamma(2/k + 1) - \Gamma^2(1/k + 1)]$. A Weibull with k>1, e.g. k=2, represents data that is more tightly grouped around its mean than is true of exponentially distributed data; it quantitatively resembles data that is log-normal. A Weibull with k<1, e.g. k=1/2, represents data of relatively extreme dispersion and positive skewness as compared to exponential data a very long right tail, compensated for by a high density near zero.

(7) Up times are iid Expon(λ). Down times are iid long-tailed log-normal h distribution. See Appendix B.

III. ANALYSIS

The methods described were simulated for various distributional assumptions which were defined in the Chapter 2. Simulation results for each method and set of distributional assumptions are shown in Table 1 and Table 2.

In general, MP has very high coverage factor and satisfactory point availability estimation. However, both average and variance of confidence length are higher than JK and LNLJ procedures.In addition, no consistent superiority of LNLJ procedure over JK procedure has been noted.

Especially under the Exponential up and Gamma (k=1/2) down, and Exponential up and Weibull (k=1/2) down times assumptions, a consistently low coverage factor and point availability estimation, and a consistently high average confidence length and variation have been obtained from LNLJ procedure. In these cases, distributional characteristics of down times after log-transform provide highly left skewed shape and large variance, and, also these features reflect in the pseudovalues which, in turn, causes left skewed shape rather than a symmetric shape for the pseudovalue distribution.

In order to compensate for this situation, the Biweight procedure, see[Ref. 7], and the Winsorizing procedure, see [Ref. 8], were implemented directly on the pseudovalues. Results are shown in Tables 3, 4, 5, 6, 7, 8. These procedures are robust/ resistant methods for establishing confidence limits on means; they tend to down-weight extreme observations that appears as outliers.

For small sample size, (i.e. n=15) Winsorizing at g=2 and g=3 levels provide an adequate coverage. But, the point availability estimate and average confidence length remain inadequate. In general, the Biweight statistical procedure caused reduction of the coverage factor, and it is not yet considered effective for compensation.

Next, a grouped jackknife procedure was applied to these cases. Simply, up and down times data were grouped two by two, and average of these groups have been taken. Obviously, this process reduced the sample sizes to one half and caused loss of degrees of freedom. However, smaller variation and more stable results were anticipated. Simulation results are presented in Tables 9 and 10. Very good coverage factors and point availability estimates were obtained, but, average confidence length remained at a rather high level. See

Appendix C for histograms of both cases for up and down times, and log-transform of down times and pseudovalues after grouped jackknife procedure applied.

Other critical simulation results were obtained from exponential up and long-tailed log-normal h down times case.For both JK and LNLJ procedures, above statistical procedures were applied, and results were shown in Tables 11, 12, 13, 14, 15, 16.In either case, after implementation of these statistical representations, the coverage factor decreased. Also, see Appendix C for histograms of this case.

Table 1: Simulation Results for Distributional Assumptions

Sample size=15

Underlying Distribution		Coverage	Average Width	Variance Width	Point Availability
A. Exponential up	JK	0.9380	0.0838	0.0022	0.9458
Log-normal down	LN	0.9443	0.0837	0.0016	0.9459
	MP	0.9698	0.0959	0.0026	0.9556
B. Exponential up	JK	0.9470	0.0850	0.0013	0.9471
Exponential down	LN	0.9417	0.1378	0.0128	0.9312
	MP	0.9868	0.1765	0.0196	0.9421
C. Exponential up	JK	0.9423	0.0706	0.0007	0.9465
Gamma down	LN	0.9419	0.0764	0.0010	0.9437
k=2	MP	0.9830	0.1001	0.0031	0.9538
D. Ditto	JK	0.9450	0.1131	0.0039	0.9442
k=1/2	LN	0.9161	0.4574	0.0932	0.8064
	MP	0.9873	0.5013	0.0970	0.8311
E. Exponential up	JK	0.9256	0.1171	0.0097	0.9437
Long-tailed h	LN	0.9483	0.1750	0.0197	0.9287
h=0.2	MP	0.9780	0.1951	0.0244	0.9412
F. Ditto	JK	0.8753	0.1663	0.0343	0.9387
h=0.4	LN	0.9406	0.2144	0.0342	0.9314
	MP	0.9513	0.2092	0.0321	0.9431

G. Long-tailed h up	JK	0.9245	0.1077	0.0024	0.9428
Exponential down	LN	0.9058	0.1650	0.0150	0.9256
h=0.2	MP	0.9803	0.1973	0.0212	0.9381
H. Ditto	JK	0.8837	0.1470	0.0048	0.9351
h=0.4	LN	0.8552	0.2100	0.0179	0.9163
	MP	0.9437	0.2377	0.0244	0.9284
I. Exponential up	JK	0.9476	0.0641	0.0005	0.9483
Weibull down	LN	0.9449	0.0672	0.0007	0.9463
k=2	MP	0.9838	0.0917	0.0023	0.9501
J. Ditto	JK	0.9142	0.1764	0.0239	0.9404
k=1/2	LN	0.9336	0.5752	0.0921	0.7649
	MP	0.9751	0.5819	0.0966	0.8464
K. Exponential up	JK	0.8968	0.1385	0.0220	0.9416
Long-tailed log	LN	0.9119	0.1217	0.0091	0.9483
normal h down	MP	0.9183	0.1113	0.0058	0.9576
h=0.2					
L. Ditto	JK	0.8794	0.1578	0.0305	0.9394
h=0.4	LN	0.9009	0.1394	0.0136	0.9483
	MP	0.9001	0.1210	0.0079	0.9575

Table 2: Simulation Results for Distributional Assumptions

Sample size=25

Underlying Distribution		Coverage	Average Width	Variance Width	Point Availability
A. Exponential up	JK	0.9436	0.0594	0.0007	0.9470
Log-normal down	LN	0.9473	0.0589	0.0004	0.9471
	MP	0.9740	0.0713	0.0008	0.9571
B. Exponential up	JK	0.9473	0.0597	0.0004	0.9471
Exponential down	LN	0.9228	0.0978	0.0052	0.9322
	MP	0.9945	0.1302	0.0086	0.9450
C. Exponential up	JK	0.9475	0.0506	0.0002	0.9474
Gamma down	LN	0.9414	0.0547	0.0003	0.9447
k=2	MP	0.9895	0.0751	0.0010	0.9552
D. Ditto	JK	0.9464	0.0766	0.0009	0.9463
k=1/2	LN	0.8187	0.4008	0.0790	0.8168
	MP	0.9884	0.4606	0.0841	0.8435
E. Exponential up	JK	0.9313	0.0834	0.0041	0.9456
Long-tailed h	LN	0.9464	0.1213	0.0077	0.9323
h=0.2	MP	0.9867	0.1426	0.0108	0.9450
F. Ditto	JK	0.8784	0.1261	0.0212	0.9421
h=0.4	LN	0.9475	0.1464	0.0132	0.9366
	MP	0.9593	0.1510	0.0139	0.9481

G.	Long-tailed h up	JK	0.9297	0.0759	0.0007	0.9454
	Exponential down	LN	0.8901	0.1167	0.0066	0.9300
	h=0.2	MP	0.9834	0.1447	0.0099	0.9422
H.	Ditto	JK	0.8802	0.1058	0.0017	0.9394
	h=0.4	LN	0.8248	0.1521	0.0073	0.9228
		MP	0.9575	0.1771	0.0116	0.9336
I.	Exponential up	JK	0.9448	0.0464	0.0002	0.9489
	Weibull down	LN	0.9433	0.0485	0.0002	0.9470
	k=2	MP	0.9879	0.0691	0.0008	0.9494
J.	Ditto	JK	0.9222	0.1188	0.0087	0.9446
	k=1/2	LN	0.8456	0.5116	0.0796	0.7765
		MP	0.9727	0.5461	0.0854	0.8245
K.	Exponential up	JK	0.9010	0.1032	0.0123	0.9440
	Long-tailed log	LN	0.9141	0.0817	0.0026	0.9507
	normal h down	MP	0.9078	0.0792	0.0018	0.9600
	h=0.2					
L.	Ditto	JK	0.8835	0.1184	0.0183	0.9426
	h=0.4	LN	0.9003	0.0918	0.0037	0.9511
		MP	0.8930	0.0848	0.0024	0.9604

Table 3: Implementation of "Winsorizing" on Exponential Up and Gamma (k=1/2) Down Times for LNLJ Procedure

Sample size=15

	Coverage	Average Width	Variance Width	Point Availability
G=0	0.9103	0.4212	0.0809	0.8239
G=1	0.9095	0.3624	0.0572	0.8686
G=2	0.9272	0.3467	0.0567	0.8840
G=3	0.9400	0.3427	0.0609	0.8954

Table 4: Implementation of "Winsorizing" on Exponential Up and Gamma (k=1/2) Down times for LNLJ Procedure

Sample size=25

	Coverage	Average Width	Variance Width	Point Availability
G=0	0.8256	0.3691	0.0670	0.8319
G=2	0.8010	0.2565	0.0265	0.8829
G=3	0.8361	0.2456	0.0247	0.8908
G=4	0.8717	0.2391	0.0247	0.8972

Table 5: Implementation of "Winsorizing" on Exponential Up and Weibull (k=1/2) Down Times for LNLJ Procedure

Sample size=15

	Coverage	Average Width	Variance Width	Point Availability
G=0	0.9336	0.5752	0.0921	0.7649
G=1	0.9418	0.5056	0.0828	0.8323
G=2	0.9526	0.4699	0.0855	0.8611
G=3	0.9510	0.4464	0.0907	0.8836

Table 6: Implementation of "Winsorizing" on Exponential Up and Weibull (k=1/2) Down Times for LNLJ Procedure

Sample size=25

	Coverage	Average Width	Variance Width	Point Availability
G=0	0.8456	0.5116	0.0796	0.7765
G=2	0.8692	0.3874	0.0516	0.8537
G=3	0.9051	0.3632	0.0513	0.8689
G=4	0.9316	0.3431	0.0511	0.8829

Table 7: Implementation of "Biweighting" on Exponential Up and Gamma (k=1/2) Down Times for LNLJ Procedure

	Sample size	Coverage	Average width	Variance width	Point Availability
Original	15	0.9103	0.4212	0.0809	0.8223
Biweighted	15	0.8935	0.2621	0.0448	0.9117
Original	25	0.8256	0.3691	0.0670	0.8319
Biweighted	25	0.8786	0.1768	0.0159	0.9195

Table 8: Implementation of "Biweighting" on Exponential Up and Weibull (k=1/2) Down Times for LNLJ Procedure

	Sample size	Coverage	Average width	Variance width	Point Availability
Original	15	0.9336	0.5752	0.0921	0.7649
Biweighted	15	0.8423	0.3163	0.0784	0.9071
Original	25	0.8456	0.5116	0.0796	0.7765
Biweighted	25	0.8374	0.2134	0.0362	0.9231

Table 9: Implementation of "Grouped Jackknife" Procedure for Exponential Up and Weibull (k=1/2) Down Times

Method	Sample Size	Coverage	Average Width	Variance Width	Point Availability
JK	14	0.9209	0.2136	0.0353	0.9381
LN	14	0.9486	0.3648	0.0774	0.9038
MP	14	0.9390	0.3083	0.0672	0.9073
JK	24	0.9223	0.1319	0.0118	0.9429
LN	24	0.9500	0.2469	0.0400	0.9148
MP	24	0.9685	0.2518	0.0452	0.9183

Table 10: Implementation of "Grouped Jackknife" Procedure for Exponential Up and Gamma (k=1/2) Down Times

Method	Sample Size	Coverage	Average Width	Variance Width	Point Availability
JK	14	0.9426	0.1380	0.0080	0.9441
LN	14	0.9534	0.2266	0.0381	0.9243
MP	14	0.9608	0.2186	0.0419	0.9262
JK	24	0.9472	0.0839	0.0015	0.9461
LN	24	0.9462	0.1448	0.0174	0.9293
MP	24	0.9783	0.1710	0.0259	0.9322

Table 11: Implementation of "Winsorizing" on Exponential Up and Long-Tailed Log-Normal (h=0.2) Down Times

Sample size=15

	Coverage	Average Width	Variance Width	Point Availability
JK				
G=0	0.8968	0.1385	0.0220	0.9416
G=1	0.8190	0.0700	0.0019	0.9544
G=2	0.7450	0.0585	0.0011	0.9560
G=3	0.7160	0.054	0.0009	0.9565
LN				
G=0	0.9119	0.1217	0.0091	0.9483
G=1	0.8560	0.0898	0.0040	0.9549
G=2	0.8040	0.0759	0.0029	0.9575
G=3	0.7460	0.0668	0.0027	0.9598

Table 12: Implementation of "Winsorizing" on Exponential Up and Long-Tailed Log-Normal (h=0.2) Down Times

Sample size=25

	Coverage	Average Width	Variance Width	Point Availability
JK				
G=0	0.9010	0.1032	0.0123	0.9440
G=2	0.7380	0.0460	0.0004	0.9554
G=3	0.6950	0.0416	0.0001	0.9563
G=4	0.6620	0.0387	0.0003	0.9568
LN				
G=0	0.9141	0.0817	0.0026	0.9507
G=2	0.8240	0.0594	0.0010	0.9558
G=3	0.7750	0.0539	0.0008	0.9573
G=4	0.7380	0.0488	0.0006	0.9588

Table 13: Implementation of "Winsorizing" on Exponential Up and Long-Tailed Log-Normal (h=0.4) Down Times

Sample size=15

	Coverage	Average Width	Variance Width	Point Availability
JK				
G=0	0.8794	0.1578	0.0305	0.9394
G=1	0.7560	0.0730	0.0026	0.9542
G=2	0.6870	0.0588	0.0015	0.9560
G=3	0.6460	0.0537	0.0012	0.9565
LN				
G=0	0.9009	0.1394	0.0136	0.9483
G=1	0.8470	0.1049	0.0069	0.9543
G=2	0.7800	0.0866	0.0053	0.9576
G=3	0.7180	0.0763	0.0048	0.9603

Table 14: Implementation of "Winsorizing" on Exponential Up and Long-Tailed Log-Normal (h=0.4) Down Times

Sample size=25

	Coverage	Average Width	Variance Width	Point Availability
JK				
G=0	0.8835	0.1184	0.0183	0.9426
G=2	0.6810	0.0473	0.0007	0.9560
G=3	0.6220	0.0411	0.0004	0.9572
G=4	0.5750	0.0380	0.0004	0.9578
LN				
G=0	0.9003	0.0918	0.0037	0.9511
G=2	0.8000	0.0666	0.0018	0.9568
G=3	0.7660	0.0585	0.0014	0.9590
G=4	0.7000	0.0531	0.0012	0.9607

Table 15: Implementation of "Biweighting" on Exponential Up and Long-Tailed Log-Normal (h=0.2) Down Times

	Sample size	Coverage	Average width	Variance width	Point Availability
JK					
Original	15	0.8968	0.1385	0.0220	0.9416
Biweighted	15	0.6870	0.0528	0.0010	0.9554
Original	25	0.9010	0.1032	0.0123	0.9440
Biweighted	25	0.6360	0.0362	0.0002	0.9569
LN					
Original	15	0.9119	0.1217	0.0091	0.9483
Biweighted	15	0.7010	0.0604	0.0019	0.9607
Original	25	0.9141	0.0817	0.0026	0.9507
Biweighted	25	0.6170	0.0406	0.0004	0.9627

Table 16: Implementation of "Biweighting" on Exponential Up and Long-Tailed Log-Normal (h=0.4) Down Times

	Sample size	Coverage	Average width	Variance width	Point Availability
JK					
Original	15	0.8794	0.1578	0.0305	0.9394
Biweighted	15	0.6210	0.0499	0.0010	0.9578
Original	25	0.8835	0.1184	0.0183	0.9426
Biweighted	25	0.5500	0.0340	0.0002	0.9590
LN					
Original	15	0.9009	0.1394	0.0136	0.9483
Biweighted	15	0.6350	0.0614	0.0025	0.9642
Original	25	0.9003	0.0918	0.0037	0.9511
Biweighted	25	0.5261	0.0394	0.0005	0.9663

IV. CONCLUSIONS

The LNLJ procedure works well when the actual down times are Exponential or Log-normal. However, the procedure is very sensitive to the sample variance. If the sample variance tends to vary greatly, sometimes becoming excessively large, then the method tends to fail; the intervals shifted down. For instance, long tailed Gamma, Weibull and Log-normal distributed down times provide extremely large sample variances, and under these circumtances the method fails.

For further study, one might apply jackknifing leaving out one observation at a time, instead leaving out a pair. Obviously, this is going to increase the degrees of freedom, but might by help reducing the variance and the confidence interval length. On the other hand, grouping more than two, might provide more stable results.

In addition, in order to see what happens in one-sided availability estimation, Exponential up and Log-normal down and Exponential up Weibull (k=1/2) down times were simulated and results are shown in Appendix D.

APPENDIX A

COMPUTER PROGRAMS

Simulation program consists of main program and several subroutines. Main program, for JK, LNLJ and MP procedures computes availability confidence limits and point availability and, scores the coverage for each replication. Then, after 1000 replications computes the statistics of these parameters and prints out the results for given underlying distributions.

Subroutine CONF computes two-sided confidence limits of a given data vector. Subroutine MOMENT computes the fourth moment of given data vector. Subroutine LNLJ generates the up and down times from given underlying distributions, and takes the log transform of these, and computes the means and the variances of these data vectors.

Subroutine BIWGT computes the Biweight estimates of a given data vector, and uses Subroutine MEDIAN for computing the median of this data. Subroutine WINSOR Winsorize a given data vector for given "g" level.

```
C*********************************************************************
C**       VARIABLE DEFINITION                                       **
C**                                                                 **
C**     U    =UP TIMES VECTOR                                       **
C**     C    =DOWN(REPAIR) TIMES VECTOR                             **
C**     X    =LOG TRANSFORM OF DOWN TIMES                           **
C**     Y    =LOG TRANSFORM OF UP TIMES                             **
C**     CBAR=SAMPLE MEAN OF UP TIMES                                **
C**     DBAR= "      "   " DOWN  "                                  **
C**     XBAR= "      "   " X VALUES                                 **
C**     YBAR= "      "   " Y VALUES                                 **
C**     XVAR= "    VARIANCE OF X VALUES                             **
C**     YVAR= "    VARIANCE OF Y VALUES                             **
C**     UMEAN=UP TIMES POPULATION MEAN                              **
C**     DMEAN=DOWN TIMES POPULATION MEAN                            **
C**     ALPHA=TYPE I ERROR                                          **
C**     CNF =CONFIDENCE                                             **
C**                                                                 **
C**       JK PROCEDURE VARIABLES                                    **
C**                                                                 **
C**     LS   =LOG(A/(1-A))=LOG(UBAR)-LOG(DBAR)                      **
C**     LSJ =J-TH VERSION OF LS                                     **
C**     PS   =PSEUDOVALUES                                          **
C**     PSBAR=MEAN OF PSEUDOVALUES                                  **
C**     S2S =VARIANCE OF  "                                         **
C**     CLS =LOWER BOUND FOR PSEUDOVALUES                           **
C**     CUS =UPPER  "     "     "                                   **
C**     AJEA=JACKKNIFE AVAILABILITY ESTIMATE                        **
C**     ALS =LOWER JACKKNIFE AVAILABILITY                           **
C**     AUS =UPPER      "          "                                **
C**     DELTA=CONFIDENCE LENGTH OF AVAILABILITY                     **
C**     AVG =AVERAGE OF DELTA                                       **
C**     VAR =VARIANCE OF DELTA                                      **
C**     SCORE=COUNTER FOR SUCCESSFUL TRIALS                         **
C**     COVER=PERCENTAGE OF SUCCESSFUL TRIALS                       **
C**                                                                 **
C**      LNLJ PROCEDURE VARIABLES                                   **
C**                                                                 **
C**     LLN =LOG(A/(1-A))=LOG(UBAR)-(XBAR+0.5*S2X)                  **
C**     XX   =MEAN OF X VECTOR OMITING J-TH VALUE                   **
C**     SSX =VARIANCE OF X VECTOR OMITING J-TH VALUE                **
C**     LLNJ=J-TH VERSION OF LLN                                    **
C**     PLN =PSEUDOVALUES                                           **
C**     PLNBAR=MEAN OF PSEUDOVALUES                                 **
C**     S2LN=VARIANCE OF   "                                        **
C**     CLLN=LOWER BOUND FOR PSEUDOVALUES                           **
C**     CULN=UPPER  "     "       "                                 **
C**     AJEB=JACKKNIFE AVAILABILITY ESTIMATE                        **
```

```
C**     ALLN=LOWER  "      "  AVAILABILITY                                   **
C**     AULN=UPPER  "      "       "                                         **
C**                                                                          **
C**     TEMPORARY VARIABLES;TEMP,A,B,C,C1,C2,SUM,SUMX,Z,ZZ                   **
C****************************************************************************
      DIMENSION U(25),D(25),X(25),Y(25),PS(25),PLN(25),DELTA1(1000),
     *          DELTA2(1000),DELTA3(1000)
      INTEGER N,RUN,IS1,IS2,I,J,K,L,IS11,IS22
      REAL Q1,Q2,Q3,Q4,Q5,Q6,Z,U,Y,X,D,UBAR,UMEAN,DMEAN,ALPHA,
     *    DBAR,XBAR,YBAR,LS,A,B,LSJ,PS,PSBAR,S2S,LLN,C,C1,SUM,SUMX,
     *    XX,C2,SSX,LLNJ,PLN,PLNBAR,S2LN,CLS,CUS,ALS,AUS,DELTA1,ALLN,
     *    SCORE1,AULN,CLLN,DELTA2,SCORE2,COVER1,COVER2,AVG1,AVG2,CNF,
     *    STD1,STD2,CULN,TEMP1,TEMP2,TEMP3,TEMP4,TEMP5,TEMP6,TEMP7,
     *    XVAR,YVAR,XM4,YM4,AVG3,DELTA3,VAR3,SESQ,PM,H
      UMEAN=73.036
      XMU=1.
      XSD=0.8325546
      H=0.4
      ALPHA=0.05
      CNF=1.-ALPHA
      L=0
      N=15
      RUN=1000
      GO TO 20
10    N=25
20    IS1=100660
      IS2=100010
      L=L+1
      Q1=0.
      Q2=0.
      Q3=0.
      Q4=0.
      Q5=0.
      Q6=0.
      TEMP6=0.
      TEMP7=0.
      TEMP8=0.
      DO 100 I=1,RUN
         CALL LNLJ(IS1,IS2,N,XMU,XSD,UMEAN,H,U,Y,D,X,UBAR,YBAR,DBAR,
     *                XBAR,YVAR,XVAR,TEMP1,TEMP2,TEMP3,IS11,IS22)
         IS1=IS11
         IS2=IS22
C
C        COMPUTE LS AND PSEUDOVALUES
C
         LS=ALOG(UBAR)-ALOG(DBAR)
         A=TEMP1
         B=TEMP2
```

```
      TEMP4=0.
      DO 103 J=1,N
         TEMP1=TEMP1-U(J)
         TEMP2=TEMP2-D(J)
         LSJ=ALOG(TEMP1)-ALOG(TEMP2)
         PS(J)=(LS*N)-(LSJ*(N-1))
         TEMP4=TEMP4+PS(J)
         TEMP1=A
         TEMP2=B
103   CONTINUE
      PSBAR=TEMP4/N
      TEMP4=0.
      DO 104 J=1,N
         TEMP4=TEMP4+(PS(J)-PSBAR)**2
104   CONTINUE
      S2S=TEMP4/(N-1)
      AJEA=(EXP(PSBAR))/(1.+EXP(PSBAR))
      TEMP6=TEMP6+AJEA
C
C     COMPUTE LLN AND PSEUDOVALUES
C
      LLN=ALOG(UBAR)-(XBAR+0.5*XVAR)
      C=TEMP3
      SUM=0.
      DO 105 J=1,N
         TEMP1=TEMP1-U(J)
         C1=X(J)
         SUMX=TEMP3-X(J)
         XX=SUMX/(N-1)
         X(J)=0.
         TEMP4=0.
         DO 106 K=1,N
            C2=(X(K)-XX)**2
            IF(X(K).EQ.0.)C2=0.
            TEMP4=TEMP4+C2
106      CONTINUE
         SSX=TEMP4/(N-2)
         TEMP5=TEMP1/(N-1)
         LLNJ=ALOG(TEMP5)-(XX+0.5*SSX)
         PLN(J)=(LLN*N)-(LLNJ*(N-1))
         SUM=SUM+PLN(J)
         TEMP1=A
         X(J)=C1
105   CONTINUE
      PLNBAR=SUM/N
      SUM=0.
      DO 107 J=1,N
         SUM=SUM+(PLN(J)-PLNBAR)**2
```

```
107         CONTINUE
            S2LN=SUM/(N-1)
            AJEB=(EXP(PLNBAR))/(1.+EXP(PLNBAR))
            GO TO 109
109         TEMP7=TEMP7+AJEB
C
C           COMPUTE CONFIDENCE LIMITS FOR JACKKNIFE PROCEDURE
C
            CALL CONF(PSBAR,S2S,N,ALPHA,CLS,CUS)
            ALS=(EXP(CLS))/(1.+EXP(CLS))
            AUS=(EXP(CUS))/(1.+EXP(CUS))
            DELTA1(I)=AUS-ALS
            SCORE1=0.
            IF(AUS.GE.CNF.AND.ALS.LE.CNF) SCORE1=1.
C
C           COMPUTE CONFIDENCE LIMITS FOR LNLJ PROCEDURE
C
            CALL CONF(PLNBAR,S2LN,N,ALPHA,CLLN,CULN)
            ALLN=(EXP(CLLN))/(1.+EXP(CLLN))
            AULN=(EXP(CULN))/(1.+EXP(CULN))
            DELTA2(I)=AULN-ALLN
            SCORE2=0.
            IF(AULN.GE.CNF.AND.ALLN.LE.CNF)SCORE2=1.
C
C           COMPUTE CONFIDENCE LIMITS FOR MOMENT PROCEDURE
C
            CALL MOMENT(X,N,XBAR,XVAR,XM4)
            CALL MOMENT(Y,N,YBAR,YVAR,YM4)
            SESQ=((YVAR+XVAR)/N)+((1./(4.*N))*(XM4+YM4-(XVAR**2)-(YVAR**2))
         *)
            PM=(YBAR+0.5*YVAR)-(XBAR+0.5*XVAR)
            AJEC=EXP(PM)/(EXP(PM)+1.)
            TEMP8=TEMP8+AJEC
            XALPHA=(1.+CNF)/2.
            CALL MDNRIS(XALPHA,YY,IER)
            CL=PM-YY*SQRT(SESQ)
            CU=PM+YY*SQRT(SESQ)
            AL=EXP(CL)/(1.+EXP(CL))
            AU=EXP(CU)/(1.+EXP(CU))
            DELTA3(I)=AU-AL
            SCORE3=0.
            IF(AU.GE.CNF.AND.AL.LE.CNF)SCORE3=1.
C
            Q1=Q1+SCORE1
            Q2=Q2+SCORE2
            Q3=Q3+SCORE3
            Q4=Q4+DELTA1(I)
            Q5=Q5+DELTA2(I)
```

```
      Q6=Q6+DELTA3(I)
100   CONTINUE
C
C     COMPUTE AND PRINT FINAL STATISTICS
C
      AJEA=TEMP6/RUN
      AJEB=TEMP7/RUN
      AJEC=TEMP8/RUN
      COVER1=Q1/RUN
      COVER2=Q2/RUN
      COVER3=Q3/RUN
      AVG1=Q4/RUN
      AVG2=Q5/RUN
      AVG3=Q6/RUN
      Q1=0.
      Q2=0.
      Q3=0.
      DO 112 I=1,RUN
         Q1=Q1+(DELTA1(I)-AVG1)**2
         Q2=Q2+(DELTA2(I)-AVG2)**2
         Q3=Q3+(DELTA3(I)-AVG3)**2
112   CONTINUE
      VAR1=Q1/(RUN-1)
      VAR2=Q2/(RUN-1)
      VAR3=Q3/(RUN-1)
      WRITE(6,7)
      WRITE(6,7)
      WRITE(6,7)
      WRITE(6,1)N,RUN
      WRITE(6,7)
      WRITE(6,2)
      WRITE(6,3)
      WRITE(6,2)
1     FORMAT(3X,'N=',I2,4X,'RUN=',I4)
2     FORMAT(1X,'********************************************')
3     FORMAT(1X,'* JACKKNIFE PROCEDURE STATISTICS RESULTS  *')
      WRITE(6,4) COVER1
      WRITE(6,5)AVG1,VAR1
      WRITE(6,7)
      WRITE(6,8)AJEA
      WRITE(6,7)
      WRITE(6,7)
      WRITE(6,2)
      WRITE(6,6)
      WRITE(6,2)
      WRITE(6,4)COVER2
      WRITE(6,5)AVG2,VAR2
      WRITE(6,7)
```

```
      WRITE(6,8)AJEB
      WRITE(6,7)
      WRITE(6,7)
      WRITE(6,2)
      WRITE(6,9)
      WRITE(6,2)
      WRITE(6,4)COVER3
      WRITE(6,5)AVG3,VAR3
      WRITE(6,7)
      WRITE(6,8)AJEC
      IF(L.EQ.2)GO TO 199
      WRITE(6,11)
      GO TO 10
4     FORMAT('0',3X,'COVERAGE IS ',F6.4)
5     FORMAT('0',3X,'AVERAGE OF DELTA IS ',F6.4,' VARIANCE IS ',F6.4)
6     FORMAT(1X,'* LNLJ PROCEDURE STATISTICS RESULTS          *')
7     FORMAT(' ')
8     FORMAT(3X,'POINT AVAILABILITY IS ',F6.4)
9     FORMAT(1X,'* MOMENT PROCEDURE STATISTICS RESULTS        *')
11    FORMAT('1')
199   STOP
      END
C*************************************************************************
C**   SUBROUTINE FOR CONFIDENCE LIMITS OF PSEUDOVALUE PARAMETERS       **
C*************************************************************************
      SUBROUTINE CONF(ZBAR,ZVAR,N,ALPHA,LB,UB)
      INTEGER N,IER
      REAL ZBAR,ZVAR,ALPHA,B,T,LB,UB
      B=FLOAT(N-1)
      CALL MDSTI(ALPHA,B,T,IER)
      LB=ZBAR-(T*SQRT(ZVAR/N))
      UB=ZBAR+(T*SQRT(ZVAR/N))
      RETURN
      END
C*************************************************************************
C**     SUBROUTINE MOMENT COMPUTES THE FOURTH MOMENT OF A DATA VECTOR.**
C*************************************************************************
      SUBROUTINE MOMENT(X,N,MEAN,VAR,M4)
      INTEGER N
      REAL X(N),MEAN,VAR,M4
      AN=N
      SUM2=0.
      DO 30 J=1,N
         DEV=X(J)-MEAN
         SUM2=SUM2+DEV**4
30    CONTINUE
      M4=SUM2*((AN-2.)*AN+3.)/((AN-1.)*(AN-2.)*(AN-3.))
      M4=M4-VAR*VAR*3.*(AN-1.)*(2.*AN-3.)/(AN*(AN-2.)*(AN-3.))
```

```
199   RETURN
      END
C*****************************************************************************
      SUBROUTINE LNLJ(IS1,IS2,N,XMU,XSD,UM,H,U,Y,D,X,UBAR,YBAR,DBAR,
     *                XBAR,YVAR,XVAR,SUM1,SUM2,SUM4,IS11,IS22)
C*****************************************************************************
      DIMENSION U(25),D(25),W(25),W1(25),XX(25),X(25),Y(25)
      INTEGER IS1,IS2,N,IS11,IS22
      REAL UM,U,D,X,UBAR,DBAR,XBAR,S2X,SUM1,SUM2,SUM3,SUM4,XMU,XVAR,H,
     *     XSD,YVAR,SUM5,SUM6
      SUM1=0.
      SUM2=0.
      SUM3=0.
      SUM4=0.
      DO 100 I=1,N
         CALL LEXPN(IS1,Z,1,1,0)
         U(I)=Z*UM
         Y(I)=ALOG(U(I))
         CALL LNORM(IS2,ZZ,1,1,0)
         XX(I)=XMU+(ZZ*XSD)
         W(I)=EXP(XX(I))
         W1(I)=W(I)*(1.+H*W(I))
         D(I)=3.844231*W1(I)/(3.844231+H*29.556224)
         X(I)=ALOG(D(I))
         SUM1=SUM1+U(I)
         SUM2=SUM2+D(I)
         SUM3=SUM3+Y(I)
         SUM4=SUM4+X(I)
100   CONTINUE
      UBAR=SUM1/N
      DBAR=SUM2/N
      YBAR=SUM3/N
      XBAR=SUM4/N
      SUM5=0.
      SUM6=0.
      DO 110 I=1,N
         SUM5=SUM5+(X(I)-XBAR)**2
         SUM6=SUM6+(Y(I)-YBAR)**2
110   CONTINUE
      XVAR=SUM5/(N-1)
      YVAR=SUM6/(N-1)
      IS11=IS1
      IS22=IS2
      RETURN
      END
```

```
C*****************************************************************************
C**   THIS SUBROUTINE COMPUTES THE BIWEIGHT ESTIMATE OF THE GIVEN           **
C**  DATA VECTOR,AND USING THIS ESTIMATE COMPUTES THE CONFIDENCE            **
C**  LIMITS.                                                                **
C**   ARGUMENTS:                                                            **
C**    X       ;INPUT DATA VECTOR (SIZE N)                                  **
C**    C       ;CONSTANT DIVISOR (6 OR 9)                                   **
C**    ALPHA   ;TYPE I ERROR FOR T TEST                                     **
C**    XHAT    ;BIWEIGHT ESTIMATE                                           **
C**    S2B     ;POPULATION VARIANCE ESTIMATE                                **
C**    S2BI    ;VARIANCE OF BIWEIGHT ESTIMATE XHAT                          **
C**    LB      ;LOWER BOUND ON BIWEIGHT ESTIMATE                            **
C**    UB      ;UPPER  "     "      "        "                              **
C**                                                                         **
C**   INTERNAL VARIABLES:                                                   **
C**    MAD     ;MEAN ABSOLUTE DEVIATION                                     **
C**    U       ;NORMALIZED VALUES                                           **
C**    W       ;WEIGHTS                                                     **
C**    EPS     ;CONSTANT FOR STOPPING RULE
C*****************************************************************************
      SUBROUTINE BIWGT(X,N,C,ALPHA,XHAT,S2B,S2BI,LB,UB)
      INTEGER N,I,IER
      DIMENSION X(25),X1(25),U(25),W(25),Y(25),Y1(25)
      REAL X,S,C,XHAT,XHAT1,SUM,SUM1,W,U,DIF,LB,UB,T,X1,S2BI,EPS,MAD,
     *     TOT,TOT1
      EPS=0.0001
      CALL MEDIAN(X,N,XHAT1)
      J=1
1     DC 10 I=1,N
         Y1(I)=ABS(X(I)-XHAT1)
10    CONTINUE
      CALL MEDIAN(Y1,N,S)
      SUM=0.
      SUM1=0.
      DO 15 I=1,N
         U(I)=(X(I)-XHAT1)/(C*S)
         IF(ABS(U(I)).GT.1.)U(I)=1.
         W(I)=(1.-(U(I)**2))**2
         SUM=SUM+W(I)
         SUM1=SUM1+W(I)*X(I)
15    CONTINUE
      XHAT=SUM1/SUM
      DIF=XHAT-XHAT1
      IF(ABS(DIF).LE.EPS)GO TO 30
      IF(J.GT.35)GO TO 30
      XHAT1=XHAT
      J=J+1
      GC TO 1
```

```
30    CONTINUE
      DO 40 I=1,N
         X1(I)=X(I)-XHAT
         Y(I)=ABS(X1(I))
40    CONTINUE
      CALL MEDIAN(Y,N,MAD)
      DO 45 I=1,N
         U(I)=X1(I)/(C*MAD)
         IF(ABS(U(I)).GE.1.)U(I)=1.
45    CONTINUE
      SUM=0.
      SUM1=0.
      DO 50 I=1,N
         IF(U(I).EQ.1.)GO TO 47
         TOT=(X1(I)**2)*((1.-(U(I)**2))**4)
         TOT1=(1.-5.*(U(I)**2))*(1.-(U(I)**2))
         GO TO 48
46       CONTINUE
47       TOT=0.
         TOT1=0.
48       SUM=SUM+TOT
         SUM1=SUM1+TOT1
50    CONTINUE
      S2BI=SUM/(SUM1*(SUM1-1.))
      S2B=S2BI*N
      DF=0.7*(N-1)
      CALL MDSTI(ALPHA,DF,T,IER)
      LB=XHAT-(T*SQRT(S2BI))
      UB=XHAT+(T*SQRT(S2BI))
      RETURN
      END
C***********************************************************************
C**   THIS SUBROUTINE COMPUTES THE MEDIAN OF A GIVEN DATA VECTOR.     **
C***********************************************************************
      SUBROUTINE MEDIAN(X,N,MED)
      INTEGER N,IQ1,M1
      REAL X(N),MED
      CALL PXSORT(X,1,N)
      IQ1=N/2
      M1=1-MOD(N,2)
      MED=(M1*X(IQ1)+X(IQ1+1))/(1+M1)
      RETURN
      END
```

```
C******************************************************************
C**    SUBROUTINE FOR WINSORIZING GIVEN DATA                     **
C******************************************************************
       SUBROUTINE WINSOR(ZZ,N,G,AA)
       INTEGER N,G,MO,M1,M2,M3
       REAL ZZ(N),AA(N)
       CALL PXSORT(ZZ,1,N)
       MO=G+1
       DO 1 I=1,MO
          AA(I)=ZZ(MO)
1      CONTINUE
       M1=MO+1
       M2=N-G
       DO 2 I=M1,M2
          AA(I)=ZZ(I)
2      CONTINUE
       M3=M2+1
       DO 3 I=M3,N
          AA(I)=ZZ(M2)
3      CONTINUE
       RETURN
       END
```

APPENDIX B

LONG-TAILED LOG-NORMAL DISTRIBUTION

(1) Let W be log-normal random variable which

$\ln W \sim N(\mu, \sigma^2)$. k moment of W as follows,

$E[W^k] = \exp(k\mu + 1/2 \cdot k^2 \sigma^2)$,

see [Ref. 9].

So,

$E[W] = \exp(\mu + 1/2 \cdot \sigma^2)$

$E[W^2] = \exp(2\mu + 2\sigma^2)$

(2) Create a stretched log-normal random variable

$D = c \cdot W \cdot (1 + h \cdot W)$

where c and h are constant and $0 < h < 1$.

$E[D] = c\{E[W] + h \cdot E[w^2]\}$

$E[D] = c\{\exp(\mu + 1/2 \cdot \sigma^2) + h \cdot \exp(2\mu + 2\sigma^2)\}$

$c = E[D]\ \{\exp(\mu + 1/2 \cdot \sigma^2) + h \cdot \exp(2\mu + 2\sigma^2)\}^{-1}$ (a.1)

(3) On the other and, we can write the median for which

is the function of the expected value as follows,

$M = d(0.5) = \psi E[D]$

$M = c \cdot \exp(\mu) \cdot (1 + h \cdot \exp(\mu)) = \psi E[D]$ (a.2)

where ψ is the median location factor of the new distribution. If we put equation (a.1) in equation (a.2), we obtain

$$\psi \bullet E[D] = E[D] \frac{\exp(\mu) \bullet (1+h \bullet \exp(\mu))}{\exp(\mu+1/2 \bullet \sigma^2) + h \bullet \exp(2\mu+2\sigma^2)}$$

After the cancellations, we obtain

$$\psi \{\exp(\mu + 1/2 \bullet \sigma^2) + h \bullet \exp(2\mu + 2\sigma^2)\} = 1+h \bullet \exp(\mu)$$

$$\psi \bullet \exp(1/2\, \sigma^2) - 1 = h \bullet \exp(\mu) \bullet (1 - \psi \bullet \exp(2\sigma^2))$$

$$h = \exp(-\mu) \frac{1 - \psi \bullet \exp(1/2 \bullet \sigma^2)}{\psi \bullet \exp(2\sigma^2) - 1}$$

If $h=0$, then $\psi = \exp(-1/2 \bullet \sigma^2)$

$$\text{if } h=1, \text{ then } \psi = \frac{1 + \exp(\mu)}{\exp(\mu+2\sigma^2) + \exp(1/2 \bullet \sigma^2)}$$

So, if we get

$$\exp(-1/2 \bullet \sigma^2) \leq \psi \leq \exp(\mu+2\sigma^2) + \exp(1/2 \bullet \sigma^2)$$

then we obtain h between 0 and 1.

APPENDIX C

DISTRIBUTIONAL FIGURES

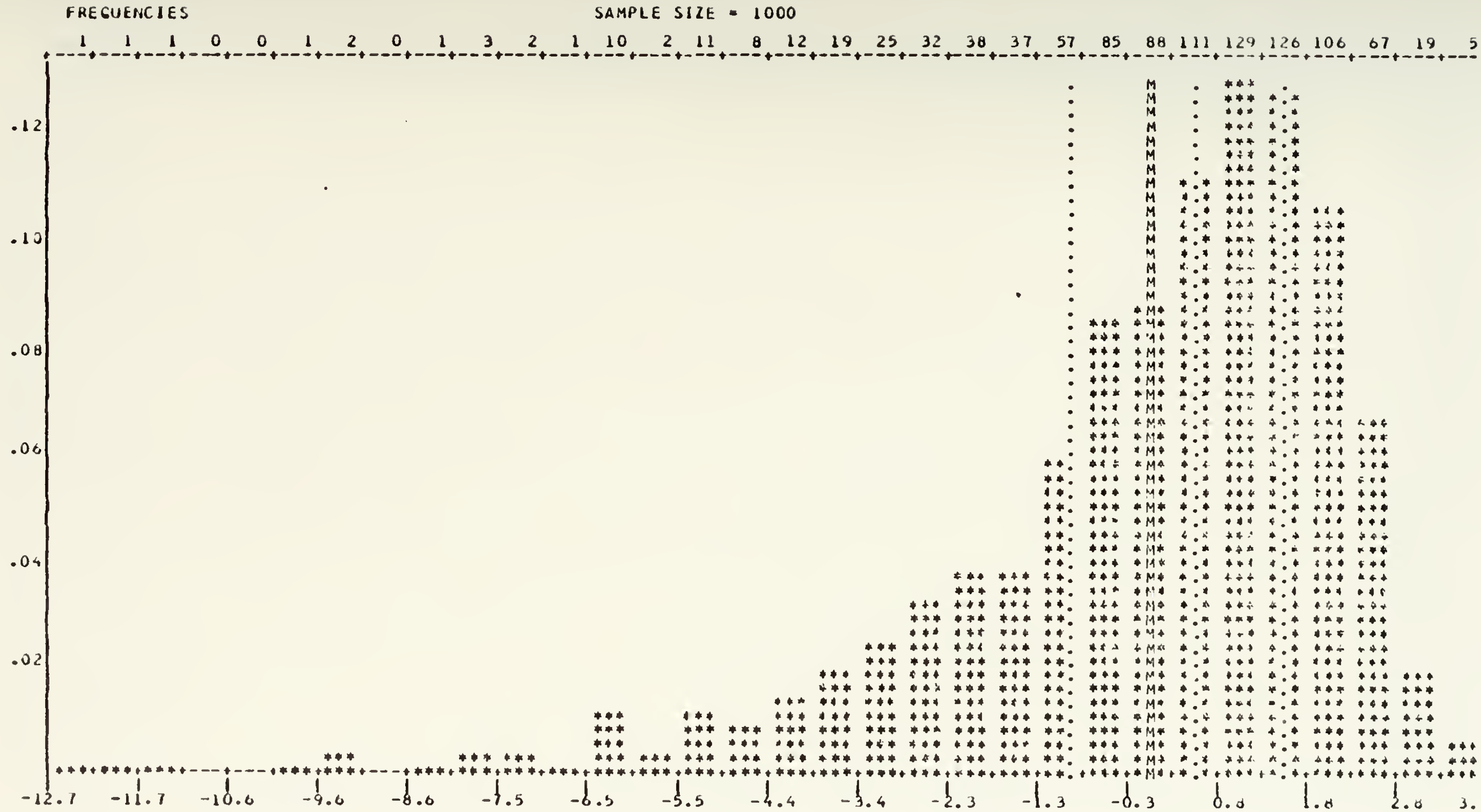
The LNLJ method fails for Exponential up and Gamma (k=1/2) down, Exponential up and Weibull (k=1/2) down, and Exponential up and Long-tailed Log- normal h down times cases. In order to demonsrate the distributional charecteristics of this cases, histograms are obtained from simulation for up times, down times, log transformations of down times and pseudovalues. Also, histograms are repeated after grouping the observations in order to observe the grouping effect on the parameters. Distributional charecteristics of these parameters such as means, variances, moments and shapes can be observed from these histograms.

CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION	
MEAN	7.200362E+01	VARIANCE	5.014352E+03	M3	6.931284E+05	MINIMUM	6.607413E-02
MEDIAN	5.258226E+01	STD DEV	7.081209E+01	M4	2.319684E+08	.10 QUANTILE	6.614846E+00
TRIMEAN	5.557187E+01	COEF VAR	9.834518E-01	SKEWNESS	1.952052E+00	.25 QUANTILE (HINGE)	1.960347E+01
MIDMEAN	5.447593E+01	MEAN DEV	4.987788E+01	KURTOSIS	6.225703E+00	.50 QUANTILE (MEDIAN)	5.258228E+01
MIDRANGE	3.116970E+02	RANGE	6.232620E+02	BETA1	6.910506E+05	.75 QUANTILE (HINGE)	9.751447E+01
GEOM MEAN	3.946925E+01	MIDSPREAD	7.790599E+01	BETA2	2.311912E+08	.90 QUANTILE	1.621982E+02
HARM MEAN	9.710757E+00					MAXIMUM	6.233281E+02

1. Exponential Up times.

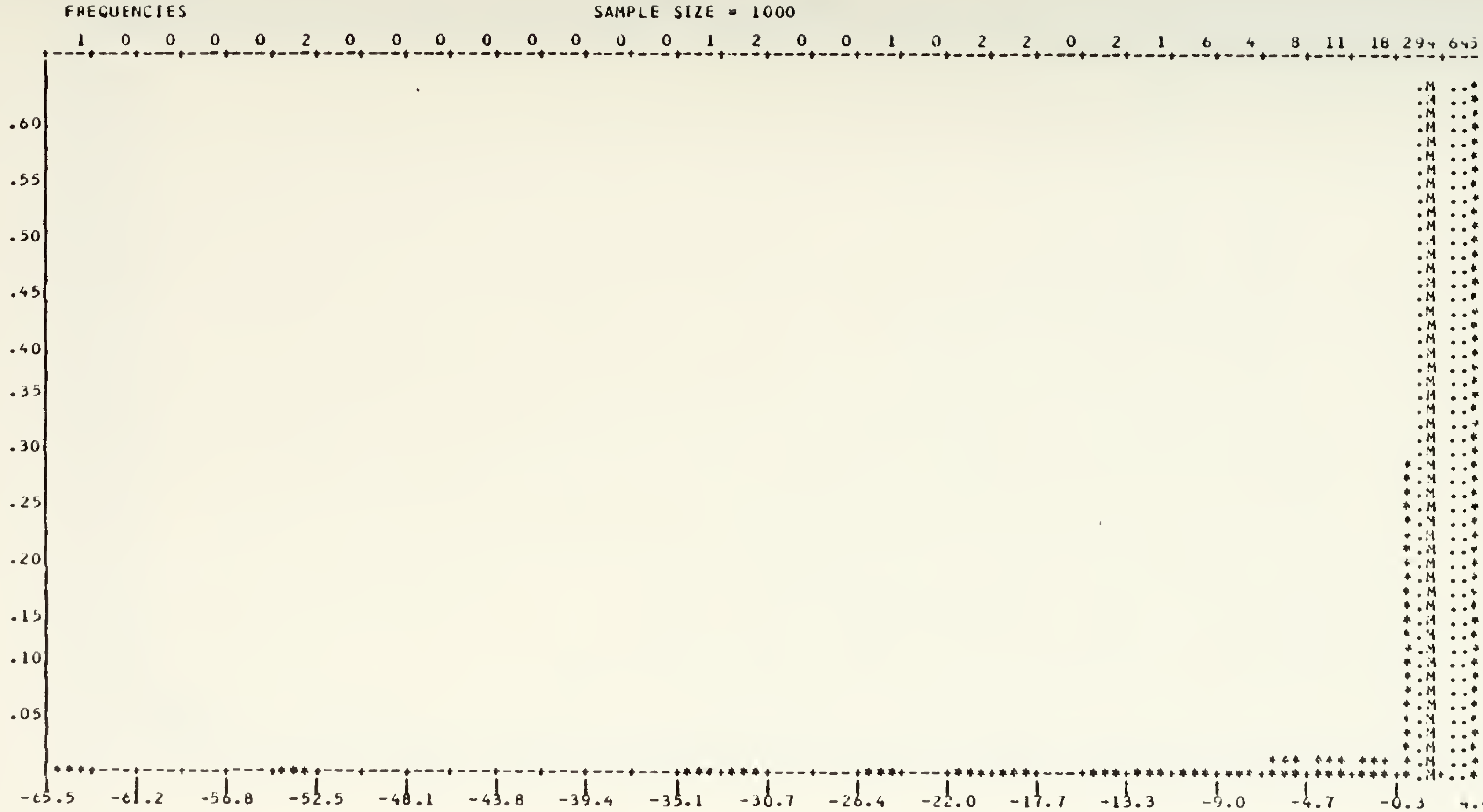
CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION	
MEAN	3.582174E+00	VARIANCE	2.448596E+01	M3	3.542698E+02	MINIMUM	2.991932E-06
MEDIAN	1.725145E+00	STD DEV	4.948329E+00	M4	9.855121E+03	.10 QUANTILE	5.632807E-02
TRIMEAN	2.211419E+00	COEF VAR	1.381375E+00	SKEWNESS	2.923874E+00	.25 QUANTILE (HINGE)	4.021568E-01
MIDMEAN	2.009779E+00	MEAN DEV	3.042477E+00	KURTOSIS	1.343718E+01	.50 QUANTILE (MEDIAN)	1.725145E+00
MIDRANGE	2.401549E+01	RANGE	4.803096E+01	BETA1	3.532080E+02	.75 QUANTILE (HINGE)	4.993231E+00
GEOM MEAN	1.038434E+00	MIDSPREAD	4.591074E+00	BETA2	9.819293E+03	.90 QUANTILE	9.333723E+00
HARM MEAN	1.541501E-03					MAXIMUM	4.803098E+01

2. Gamma (k=1/2) down times.



CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION		
MEAN	3.771428E-02	VARIANCE	4.896603E+00	M3	-1.702757E+01	MINIMUM		-1.271959E+01
MEDIAN	5.453115E-01	STD DEV	2.212827E+00	M4	1.651129E+02	.10 QUANTILE		-2.876362E+00
TRIMEAN	4.469476E-01	COEF VAR	5.867343E+01	SKEWNESS	-1.571486E+00	.25 QUANTILE	(HINGE)	-9.109135E-01
MIDMEAN	4.743037E-01	MEAN DEV	1.602086E+00	KURTOSIS	3.886385E+00	.50 QUANTILE	(MEDIAN)	5.453115E-01
MIDRANGE	-4.423873E+00	RANGE	1.659143E+01	BETA1	-1.697653E+01	.75 QUANTILE	(HINGE)	1.608081E+00
		MIDSPREAD	2.518994E+00	BETA2	1.645961E+02	.90 QUANTILE		2.233634E+00
						MAXIMUM		3.871846E+00

3. Log-transforms of Gamma (k = 1/2) down times.



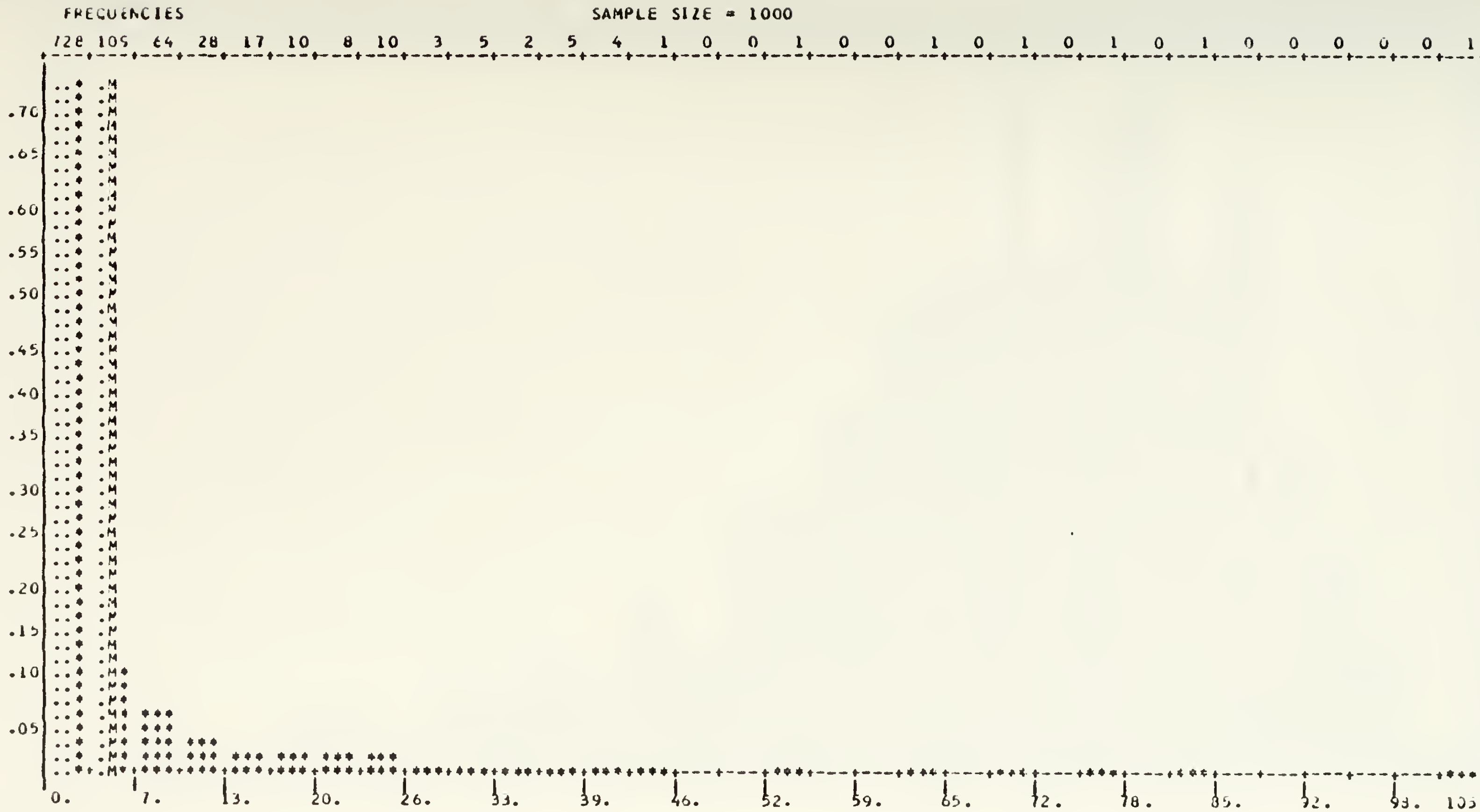
CENTRAL TENDENCY	
MEAN	1.756527E+00
MEDIAN	2.722046E+00
TRIMEAN	2.643311E+00
MIDMEAN	2.617613E+00
MIDRANGE	-3.072900E+01

SPREAD	
VARIANCE	2.195885E+01
STD DEV	4.686027E+00
COEF VAR	2.608380E+00
MEAN DEV	1.764929E+00
RANGE	6.953809E+01
MIDSPREAD	2.223145E+00

HIGHER CENTRAL MOMENTS	
M3	-8.732380E+02
M4	4.649375E+04
SKEWNESS	-8.486306E+00
KURTOSIS	9.342186E+01
BETA1	-8.706201E+02
BETA2	4.631066E+04

DISTRIBUTION		
MINIMUM		-6.549905E+01
.10 QUANTILE		2.597656E-01
.25 QUANTILE	(HINGE)	1.453003E+00
.50 QUANTILE	(MEDIAN)	2.722046E+00
.75 QUANTILE	(HINGE)	3.676147E+00
.90 QUANTILE		3.982910E+00
MAXIMUM		4.040039E+00

4. Pseudovalues of LNLJ procedure for Exponential UP and Gamma (k= 1/2) down times.



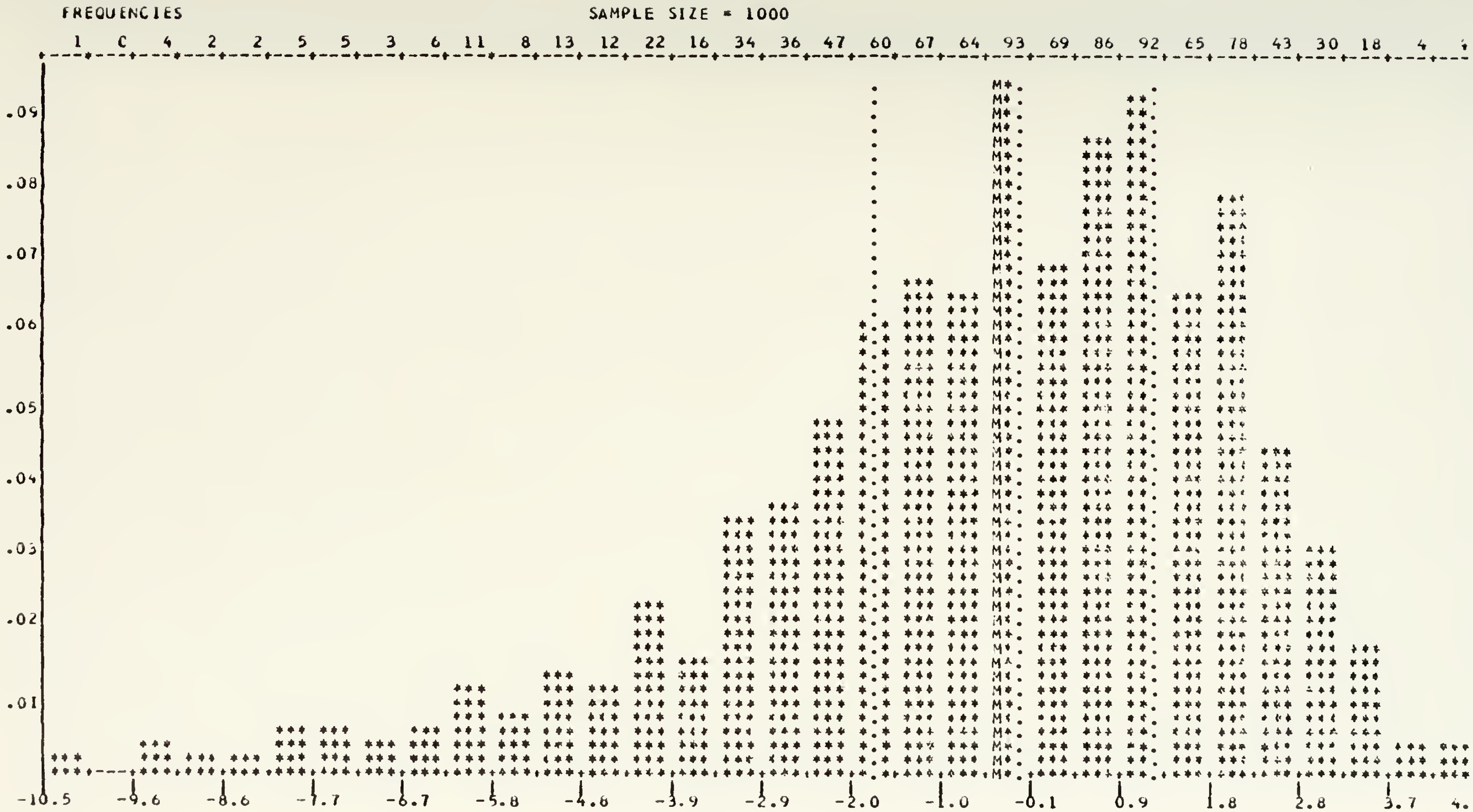
CENTRAL TENDENCY	
MEAN	3.852020E+00
MEDIAN	8.752610E-01
TRIMEAN	1.383789E+00
MIDMEAN	1.230354E+00
MIDRANGE	5.231058E+01
GEOM MEAN	6.335018E-01
HARM MEAN	6.674167E-03

SPREAD	
VARIANCE	7.244522E+01
STD DEV	8.511476E+00
COEF VAR	2.209613E+00
MEAN DEV	3.594235E+00
RANGE	1.046211E+02
MIDSPREAD	3.436481E+00

HIGHER CENTRAL MOMENTS	
M3	3.340087E+03
M4	2.331044E+05
SKEWNESS	5.416807E+00
KURTOSIS	4.141513E+01
BETA1	3.330075E+03
BETA2	2.322034E+05

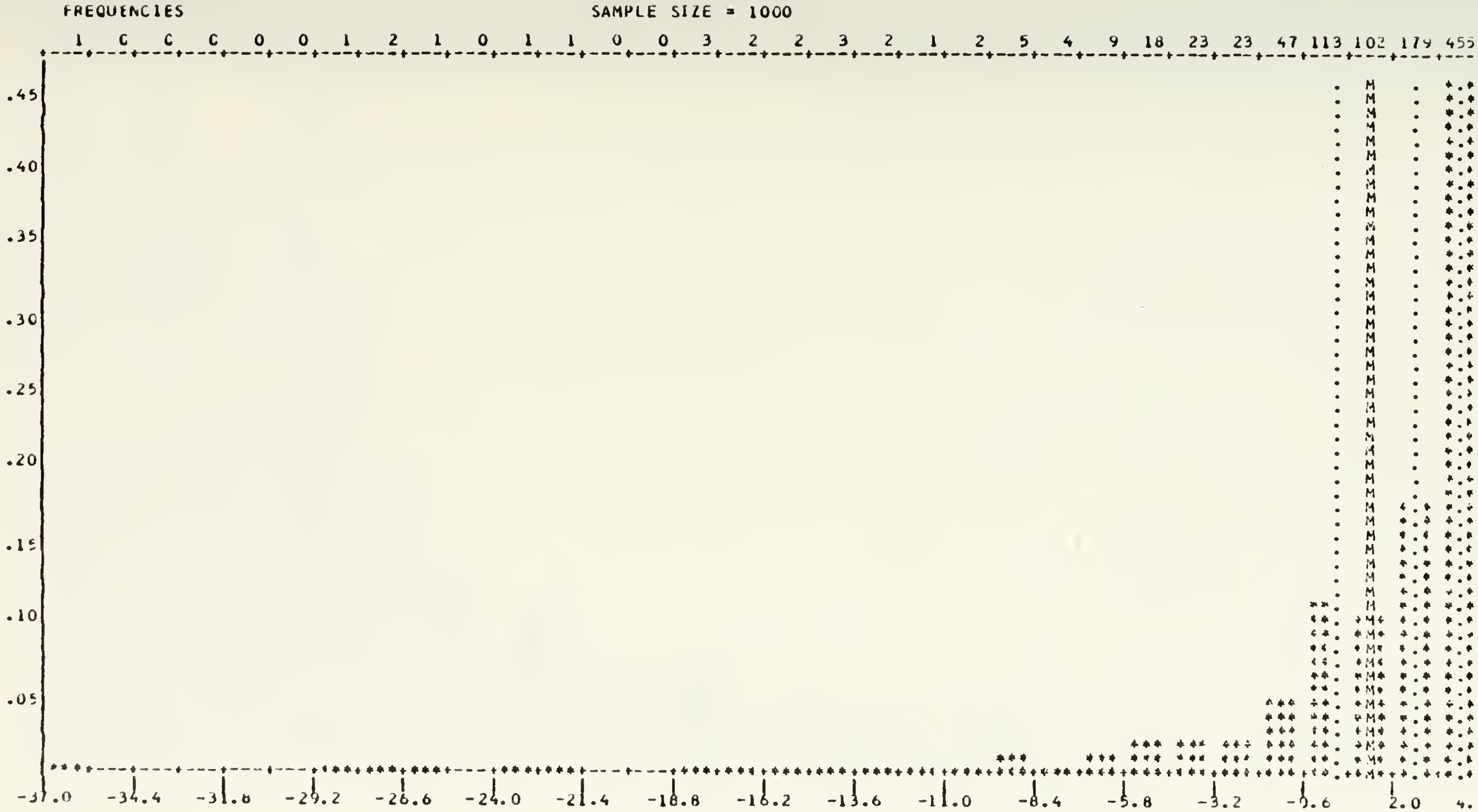
DISTRIBUTION		
MINIMUM		2.681305E-05
.10 QUANTILE		2.203226E-02
.25 QUANTILE	(HINGE)	1.700773E-01
.50 QUANTILE	(MEDIAN)	8.752610E-01
.75 QUANTILE	(HINGE)	3.606559E+00
.90 QUANTILE		9.619667E+00
MAXIMUM		1.046212E+02

5. Weibull (k = 1/2) down times.



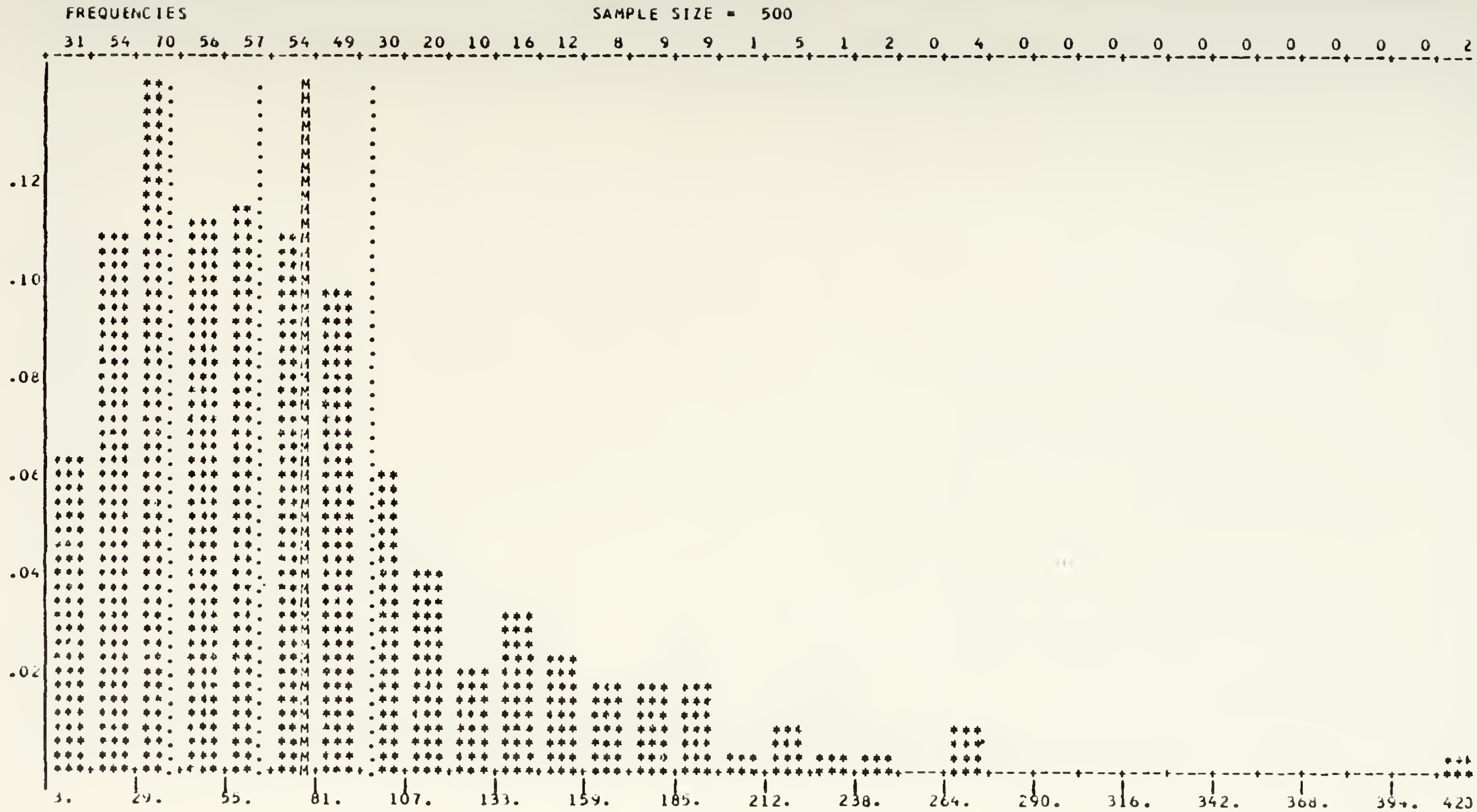
CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION	
MEAN	-4.564924E-01	VARIANCE	6.053996E+00	M3	-1.355162E+01	MINIMUM	-1.052662E+01
MEDIAN	-1.286735E-01	STD DEV	2.460487E+00	M4	1.483882E+02	.10 QUANTILE	-3.815248E+00
TRIMEAN	-1.665249E-01	COEF VAR	5.389985E+00	SKEWNESS	-9.097620E-01	.25 QUANTILE (HINGE)	-1.771502E+00
MIDMEAN	-1.382045E-01	MEAN DEV	1.888613E+00	KURTOSIS	1.048695E+00	.50 QUANTILE (MEDIAN)	-1.286735E-01
MIDRANGE	-2.938138E+00	RANGE	1.517697E+01	BETA1	-1.351099E+01	.75 QUANTILE (HINGE)	1.282748E+00
		MIDSPREAD	3.054251E+00	BETA2	1.480142E+02	.90 QUANTILE	2.263809E+00
						MAXIMUM	4.650346E+00

6. Log-transform of Weibull (k = 1/2) down times.



CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION	
MEAN	1.697203E+00	VARIANCE	1.741412E+01	M3	-2.765720E+02	MINIMUM	-3.698633E+01
MEDIAN	2.992676E+00	STD DEV	4.173023E+00	M4	7.315977E+03	.10 QUANTILE	-2.205322E+00
TRIMEAN	2.698486E+00	COEF VAR	2.458765E+00	SKEWNESS	-3.805890E+00	.25 QUANTILE (HINGE)	5.712891E-01
MIDMEAN	2.836321E+00	MEAN DEV	2.382576E+00	KURTOSIS	2.112511E+01	.50 QUANTILE (MEDIAN)	2.992576E+00
MIDRANGE	-1.619897E+01	RANGE	4.157471E+01	BETA1	-2.757432E+02	.75 QUANTILE (HINGE)	4.237305E+00
		MIDSPREAD	3.666016E+00	BETA2	7.288527E+03	.90 QUANTILE	4.499023E+00
						MAXIMUM	4.588379E+00

7. Pseudovalues of LNLJ procedure for Exponential up and Weibull (k = 1/2) down times.



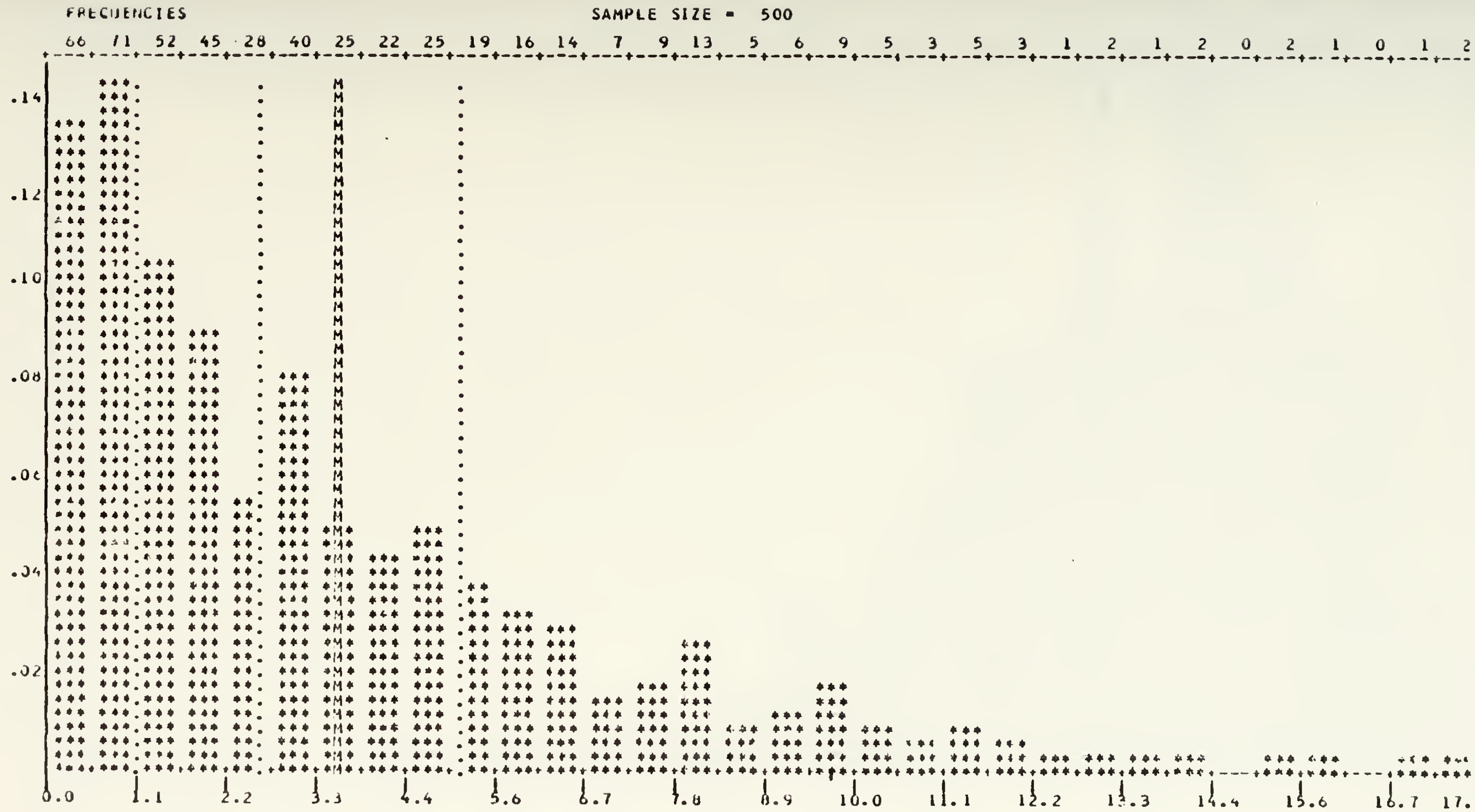
CENTRAL TENDENCY	
MEAN	7.521086E+01
MEDIAN	6.332124E+01
TRIMEAN	6.452688E+01
MIDMEAN	6.379004E+01
MIDRANGE	2.115084E+02
GEOM MEAN	5.754243E+01
HARM MEAN	3.946172E+01

SPREAD	
VARIANCE	3.089728E+03
STD DEV	5.558531E+01
COEF VAR	7.390596E-01
MEAN DEV	3.933289E+01
RANGE	4.177939E+02
MIDSPREAD	5.891000E+01

HIGHER CENTRAL MOMENTS	
M3	3.232222E+05
M4	8.601296E+07
SKEWNESS	1.882007E+00
KURTOSIS	6.009974E+00
BETA1	3.212855E+05
BETA2	8.543907E+07

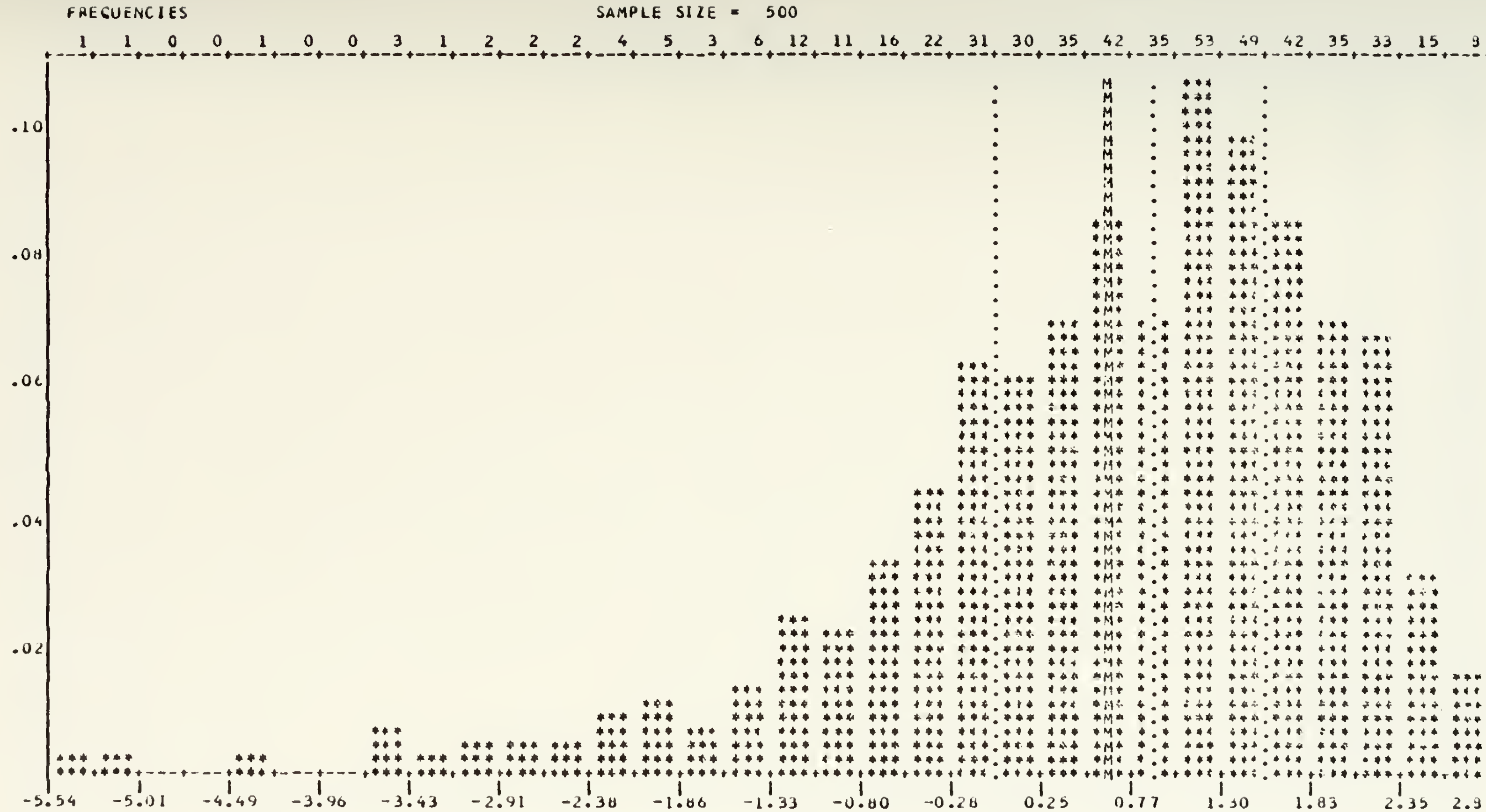
DISTRIBUTION	
MINIMUM	2.611371E+00
.10 QUANTILE	2.228358E+01
.25 QUANTILE (HINGE)	3.707762E+01
.50 QUANTILE (MEDIAN)	6.332124E+01
.75 QUANTILE (HINGE)	9.598763E+01
.90 QUANTILE	1.471443E+02
MAXIMUM	4.204055E+02

8. Exponential up times after grouping.



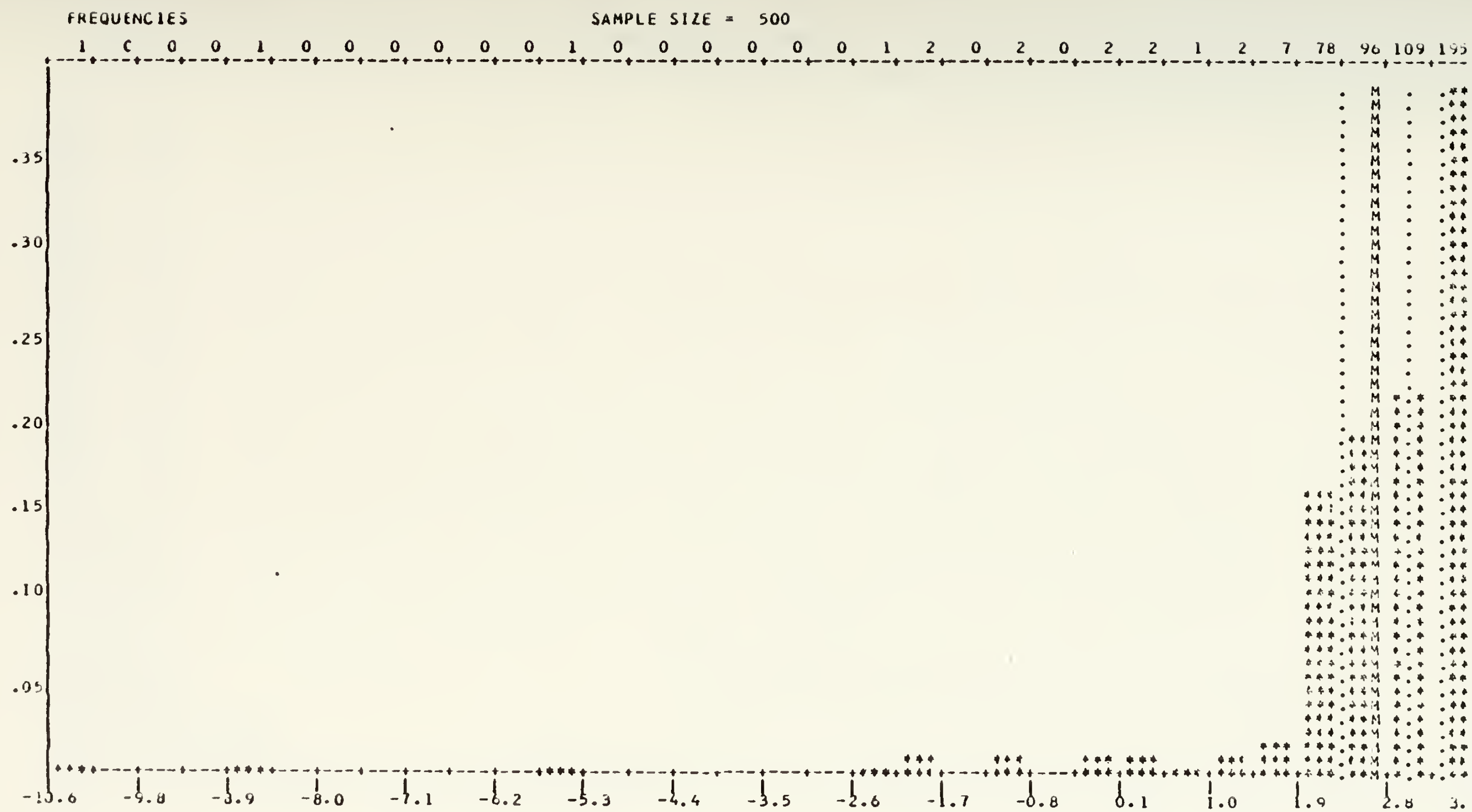
CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION		
MEAN	3.540178E+00	VARIANCE	1.119337E+01	M3	5.600957E+01	MINIMUM		3.936958E-03
MEDIAN	2.601983E+00	STD DEV	3.345650E+00	M4	6.618396E+02	.10 QUANTILE		3.823637E-01
TRIMEAN	2.817018E+00	COEF VAR	9.450511E-01	SKEWNESS	1.495618E+00	.25 QUANTILE	(HINGE)	9.966145E-01
MIDMEAN	2.665813E+00	MEAN DEV	2.448792E+00	KURTOSIS	2.282393E+00	.50 QUANTILE	(MEDIAN)	2.601983E+00
MIDRANGE	8.891556E+00	RANGE	1.777524E+01	BETA1	5.567397E+01	.75 QUANTILE	(HINGE)	5.067492E+00
GEOM MEAN	2.017651E+00	MIDSPREAD	4.070877E+00	BETA2	6.580439E+02	.90 QUANTILE		8.235460E+00
HARM MEAN	4.318705E-01					MAXIMUM		1.777917E+01

9. Gamma (k = 1/2) down times after grouping.



CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION		
MEAN	7.019536E-01	VARIANCE	1.639483E+00	M3	-2.526958E+00	MINIMUM		-5.537347E+00
MEDIAN	9.562736E-01	STD DEV	1.280423E+00	M4	1.460977E+01	.10 QUANTILE		-9.613832E-01
TRIMEAN	8.829863E-01	COEF VAR	1.824084E+00	SKEWNESS	-1.203753E+00	.25 QUANTILE	(HINGE)	-3.392745E-03
MIDMEAN	8.794547E-01	MEAN DEV	9.697692E-01	KURTOSIS	2.435373E+00	.50 QUANTILE	(MEDIAN)	9.562736E-01
MIDRANGE	-1.329659E+00	RANGE	8.415375E+00	BETA1	-2.511817E+00	.75 QUANTILE	(HINGE)	1.622791E+00
		MIDSPREAD	1.626184E+00	BETA2	1.452505E+01	.90 QUANTILE		2.108449E+00
						MAXIMUM		2.878028E+00

10. Log-transform of Gamma (k = 1/2) after grouping.



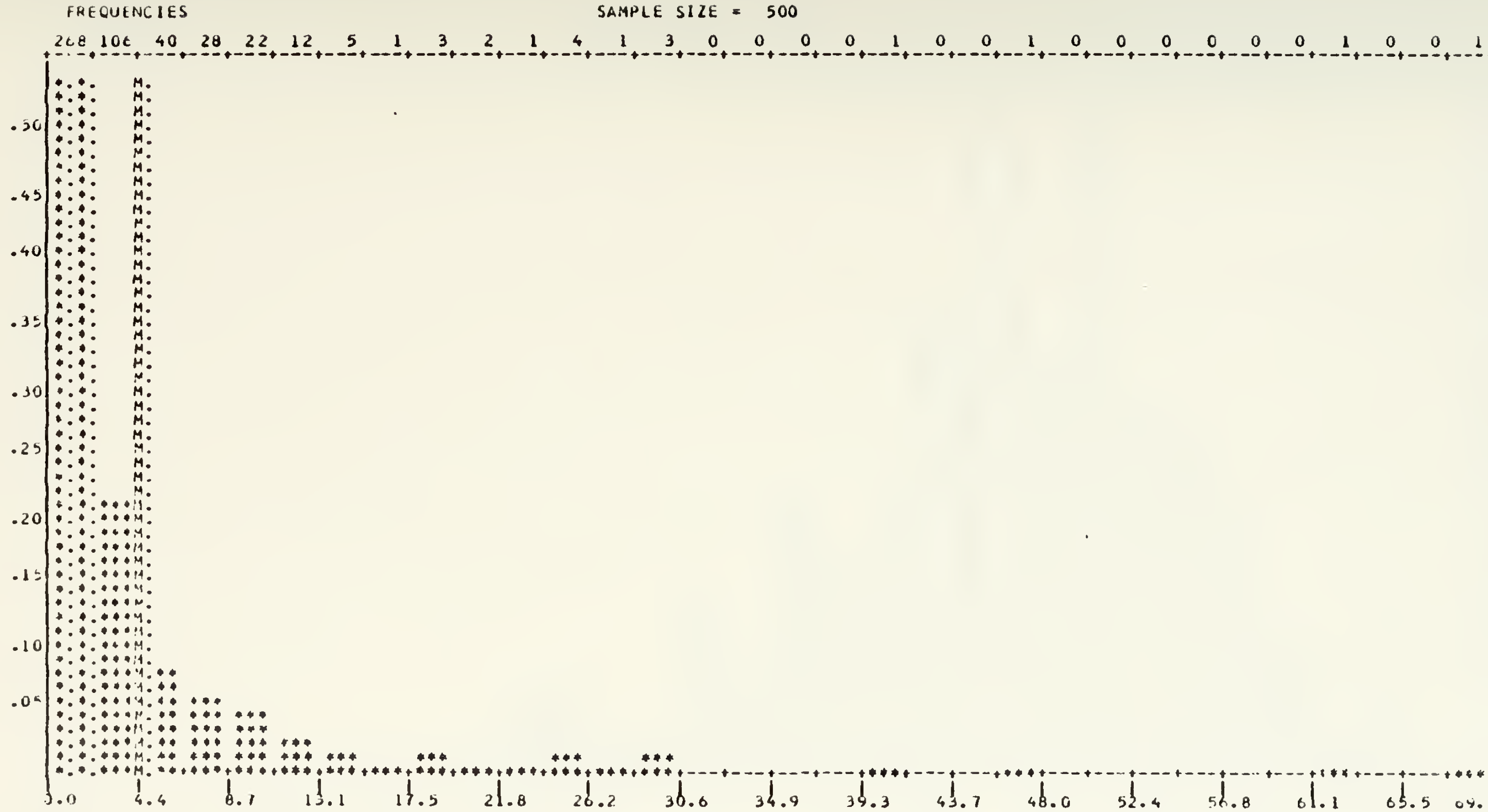
CENTRAL TENDENCY	
MEAN	2.798450E+00
MEDIAN	3.067017E+00
TRIMEAN	2.995483E+00
MIDMEAN	3.003094E+00
MIDRANGE	-3.486694E+00

SPREAD	
VARIANCE	1.328687E+00
STD DEV	1.152687E+00
COEF VAR	4.119019E-01
MEAN DEV	5.853125E-01
RANGE	1.432446E+01
MIDSPREAD	9.982910E-01

HIGHER CENTRAL MOMENTS	
M3	-1.012170E+01
M4	1.148537E+02
SKEWNESS	-6.608752E+00
KURTOSIS	6.205786E+01
BETA1	-1.006106E+01
BETA2	1.139560E+02

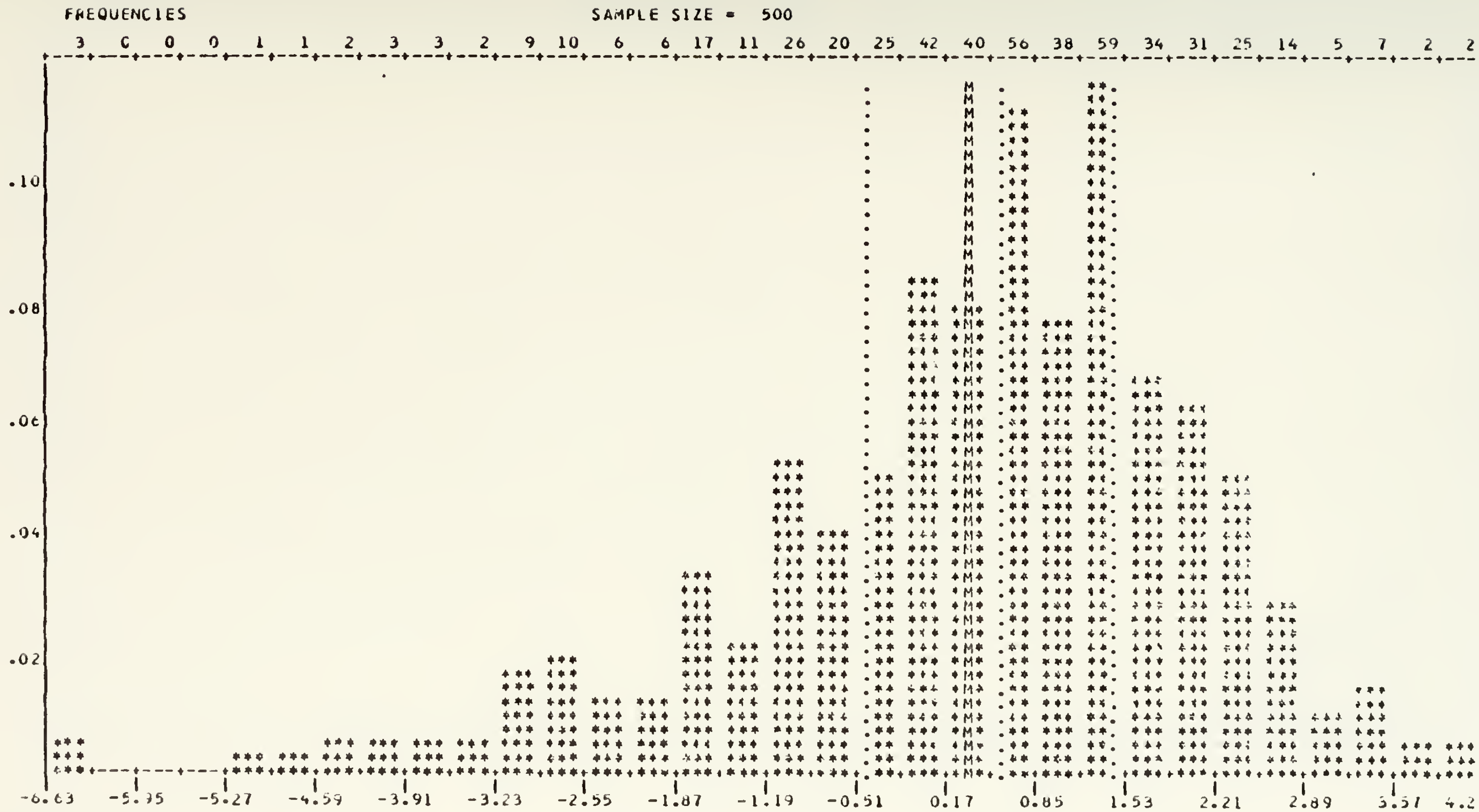
DISTRIBUTION		
MINIMUM		-1.064893E+01
.10 QUANTILE		2.113281E+00
.25 QUANTILE	(HINGE)	2.424805E+00
.50 QUANTILE	(MEDIAN)	3.067017E+00
.75 QUANTILE	(HINGE)	3.423096E+00
.90 QUANTILE		3.539551E+00
MAXIMUM		3.675537E+00

11. Pseudovalues of LNLJ procedure for Exponential Up and Gamma (k = 1/2) down times after grouping.



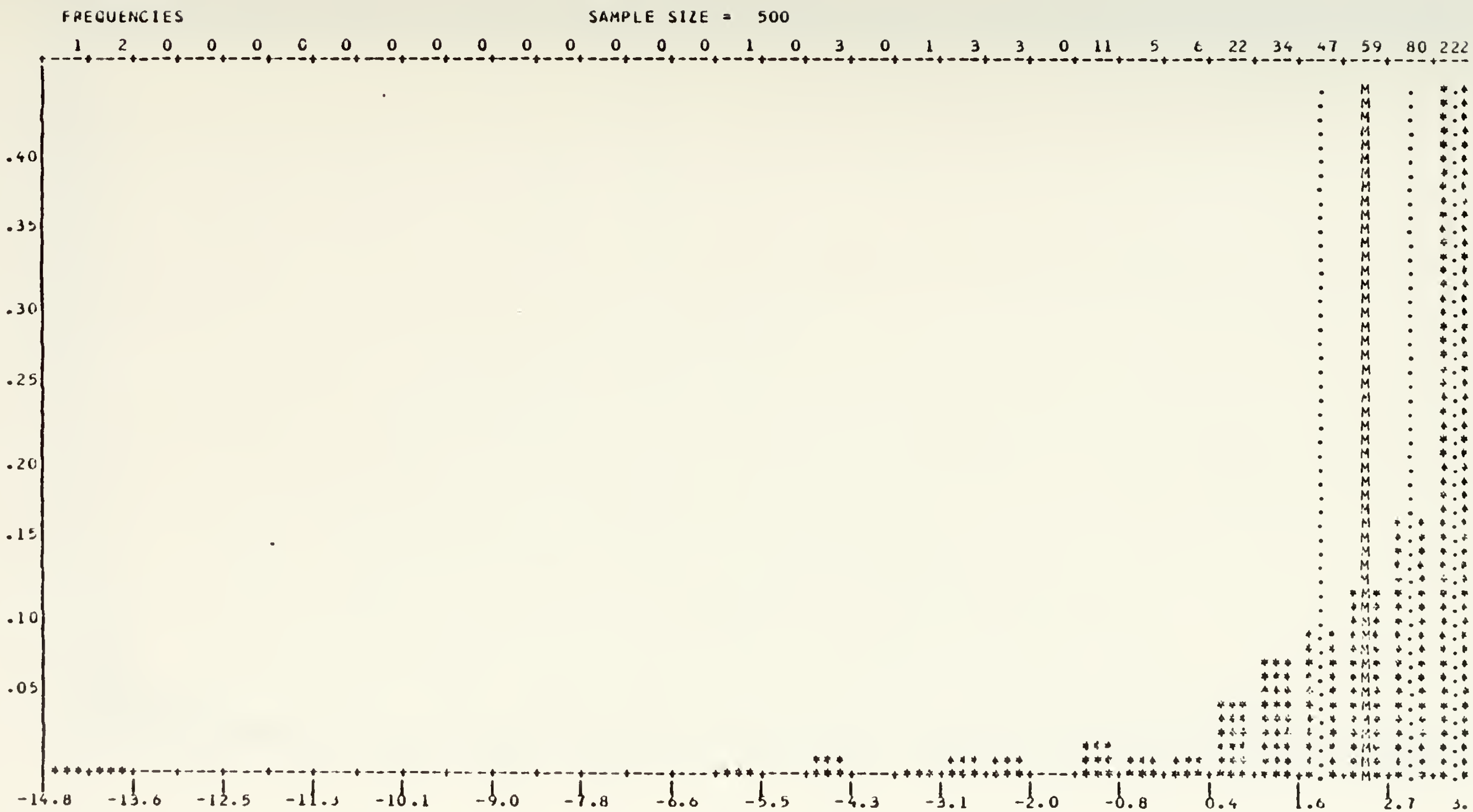
CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION	
MEAN	3.990130E+00	VARIANCE	4.463084E+01	M3	1.494118E+03	MINIMUM	1.326655E-03
MEDIAN	1.929276E+00	STD DEV	6.680632E+00	M4	7.687900E+04	.10 QUANTILE	1.702547E-01
TRIMEAN	2.230384E+00	COEF VAR	1.674289E+00	SKEWNESS	5.011093E+00	.25 QUANTILE (HINGE)	6.741939E-01
MIDMEAN	2.151340E+00	MEAN DEV	3.239119E+00	KURTOSIS	3.559557E+01	.50 QUANTILE (MEDIAN)	1.929276E+00
MIDRANGE	3.493510E+01	RANGE	6.986757E+01	BETA1	1.485166E+03	.75 QUANTILE (HINGE)	4.368792E+00
GEOM MEAN	1.494725E+00	MIDSPREAD	3.714598E+00	BETA2	7.628787E+04	.90 QUANTILE	9.662323E+00
HARM MEAN	1.327060E-01					MAXIMUM	6.986890E+01

12. Weibull (k = 1/2) down times after grouping.



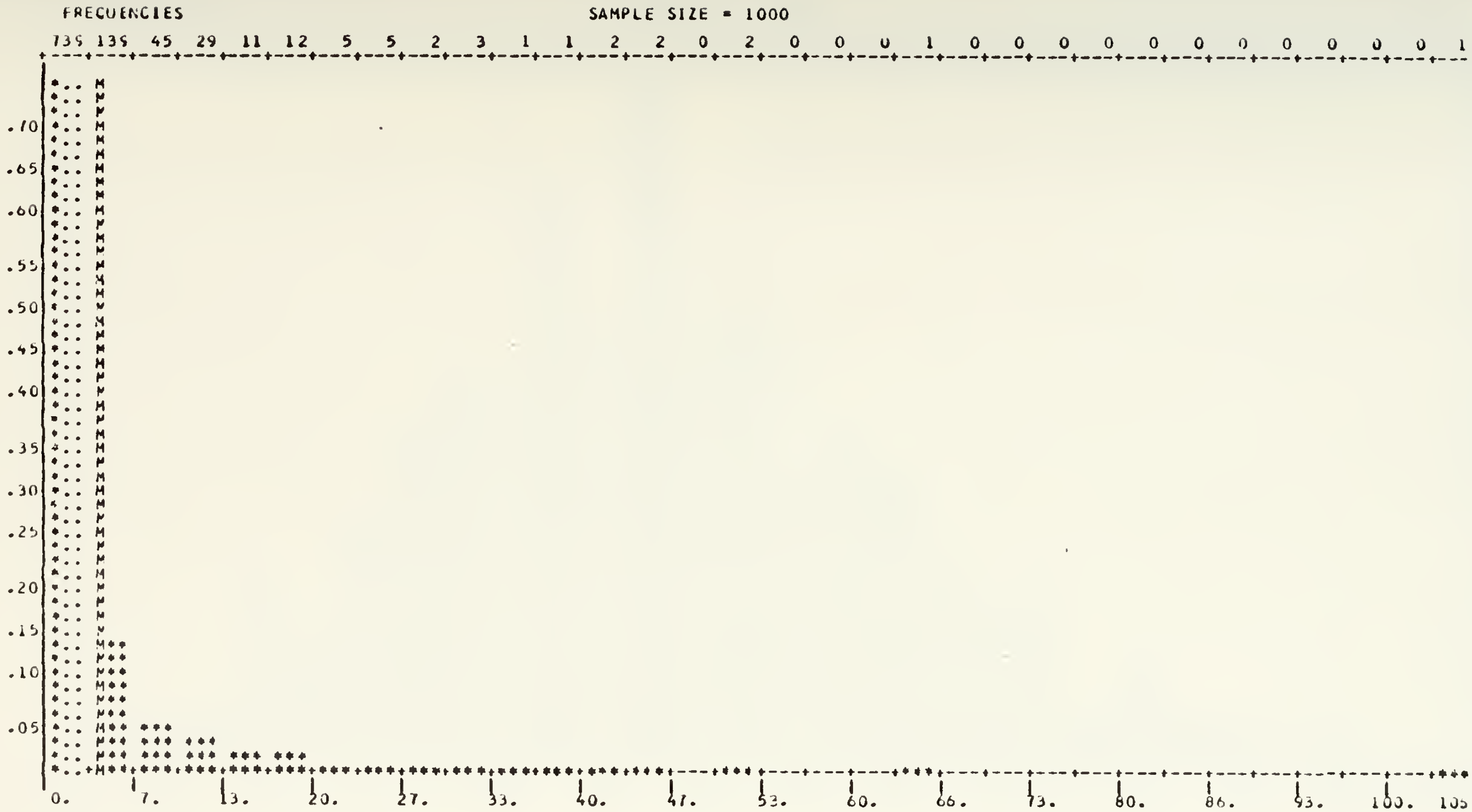
CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION	
MEAN	4.019423E-01	VARIANCE	2.815965E+00	M3	-4.587463E+00	MINIMUM	-6.625095E+00
MEDIAN	6.571255E-01	STD DEV	1.677815E+00	M4	3.639705E+01	.10 QUANTILE	-1.770459E+00
TRIMEAN	5.556957E-01	COEFF VAR	4.174270E+00	SKEWNESS	-9.712704E-01	.25 QUANTILE (HINGE)	-3.945163E-01
MIDMEAN	6.329320E-01	MEAN DEV	1.253912E+00	KURTOSIS	1.592923E+00	.50 QUANTILE (MEDIAN)	6.571255E-01
MIDRANGE	-1.189238E+00	RANGE	1.087172E+01	BETA1	-4.555976E+00	.75 QUANTILE (HINGE)	1.479049E+00
		MIDSPREAD	1.873565E+00	BETA2	3.620070E+01	.90 QUANTILE	2.268234E+00
						MAXIMUM	4.246620E+00

13. Log-transform of Weibull (k = 1/2) down times after grouping.



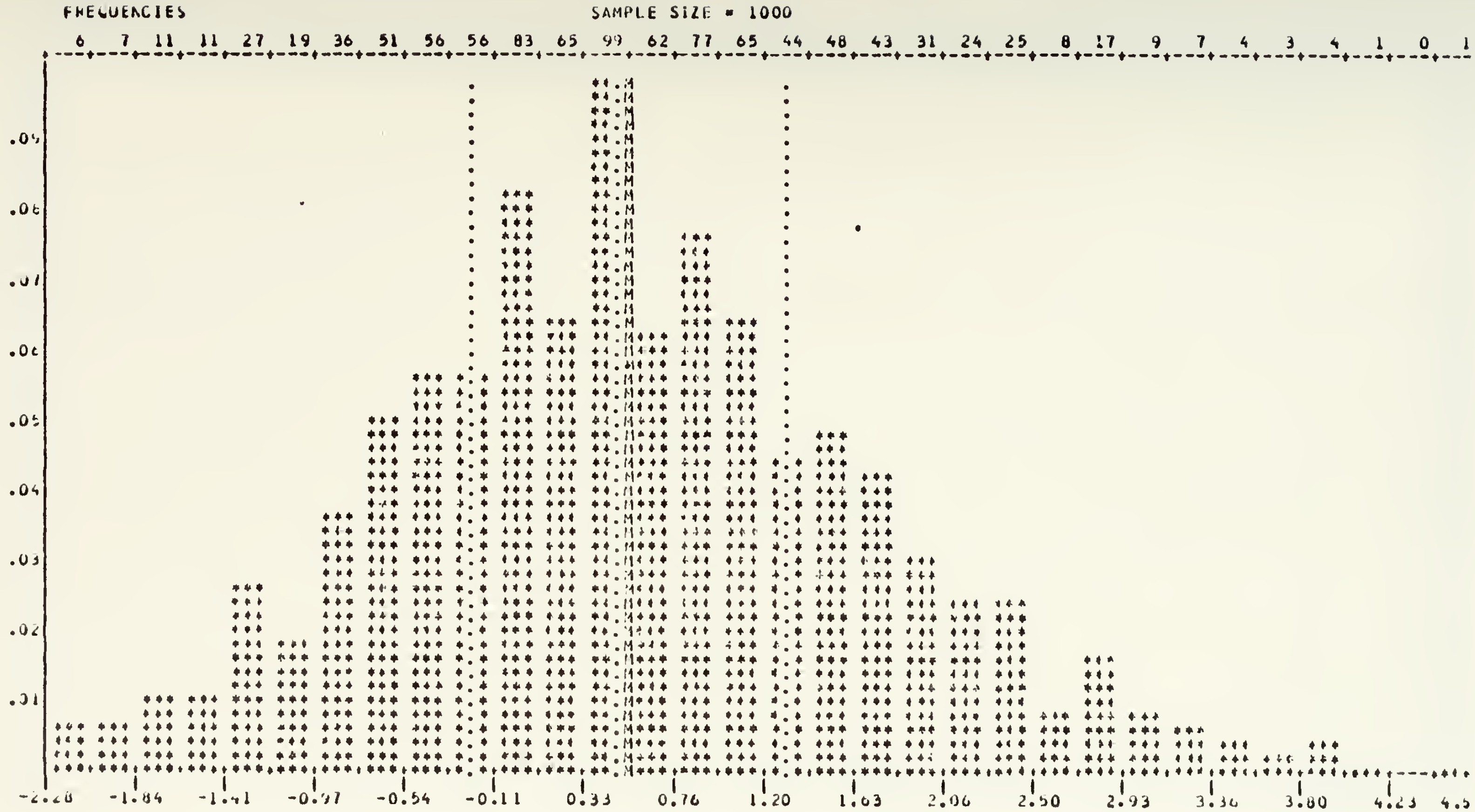
CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION	
MEAN	2.510654E+00	VARIANCE	3.897135E+00	M3	-3.509424E+01	MINIMUM	-1.480640E+01
MEDIAN	3.090332E+00	STD DEV	1.974116E+00	M4	5.281067E+02	.10 QUANTILE	8.391113E-01
TRIMEAN	2.940674E+00	COEF VAR	7.862955E-01	SKEWNESS	-4.561604E+00	.25 QUANTILE (HINGE)	1.896606E+00
MIDMEAN	3.323156E+00	MEAN DEV	1.109790E+00	KURTOSIS	3.177208E+01	.50 QUANTILE (MEDIAN)	3.090332E+00
MIDRANGE	-5.456543E+00	RANGE	1.869971E+01	BETA1	-3.488396E+01	.75 QUANTILE (HINGE)	3.685425E+00
		MIDSPREAD	1.788818E+00	BETA2	5.240637E+02	.90 QUANTILE	3.835938E+00
						MAXIMUM	3.893311E+00

14. Pseudovalues of LNLJ procedure for Exponential up and Weibull (k = 1/2) down times after grouping.



CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION	
MEAN	3.651278E+00	VARIANCE	4.671630E+01	M3	2.113687E+03	MINIMUM	1.026774E-01
MEDIAN	1.637004E+00	STD DEV	6.834969E+00	M4	1.548466E+05	.10 QUANTILE	4.142764E-01
TRIMEAN	1.924339E+00	COEF VAR	1.871938E+00	SKEWNESS	6.619596E+00	.25 QUANTILE (HINGE)	7.773933E-01
MIDMEAN	1.910139E+00	MEAN DEV	2.815007E+00	KURTOSIS	6.795053E+01	.50 QUANTILE (MEDIAN)	1.637004E+00
MIDRANGE	5.320755E+01	RANGE	1.062097E+02	BETA1	2.107352E+03	.75 QUANTILE (HINGE)	3.645519E+00
GEOM MEAN	1.753210E+00	MIDSPREAD	2.868526E+00	BETA2	1.542403E+05	.90 QUANTILE	7.944254E+00
HARM MEAN	9.641558E-01					MAXIMUM	1.063124E+02

15. Long-tailed Log-normal h down times.



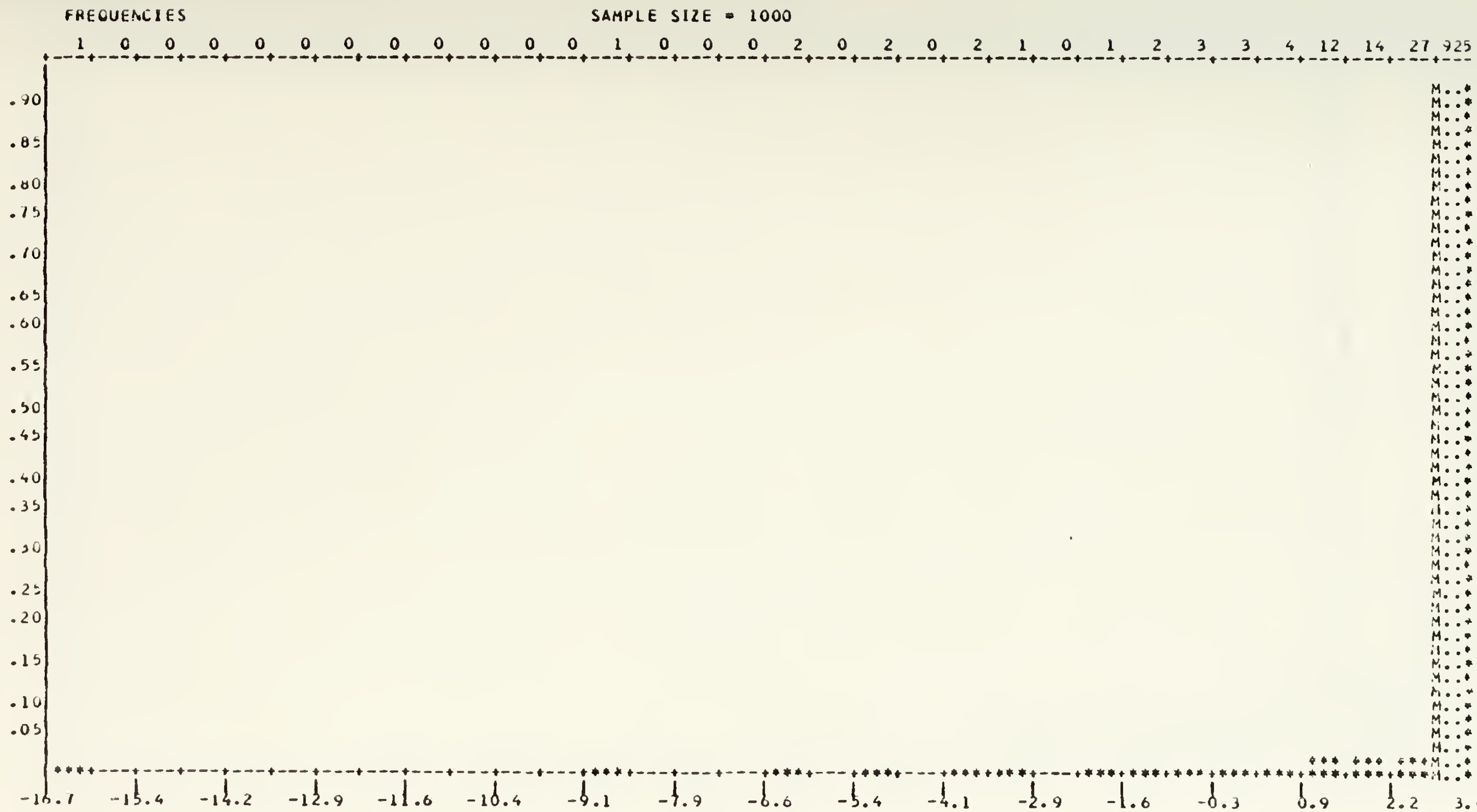
CENTRAL TENDENCY	
MEAN	5.614485E-01
MEDIAN	4.928679E-01
TRIMEAN	5.068836E-01
MIDMEAN	5.096651E-01
MIDRANGE	1.195109E+00

SPREAD	
VARIANCE	1.331922E+00
STD DEV	1.154089E+00
COEF VAR	2.055555E+00
MEAN DEV	9.077550E-01
RANGE	6.942545E+00
MIDSPREAD	1.545416E+00

HIGHER CENTRAL MOMENTS	
M3	4.400648E-01
M4	5.444527E+00
SKEWNESS	2.862854E-01
KURTOSIS	6.904221E-02
BETA1	4.387456E-01
BETA2	5.433377E+00

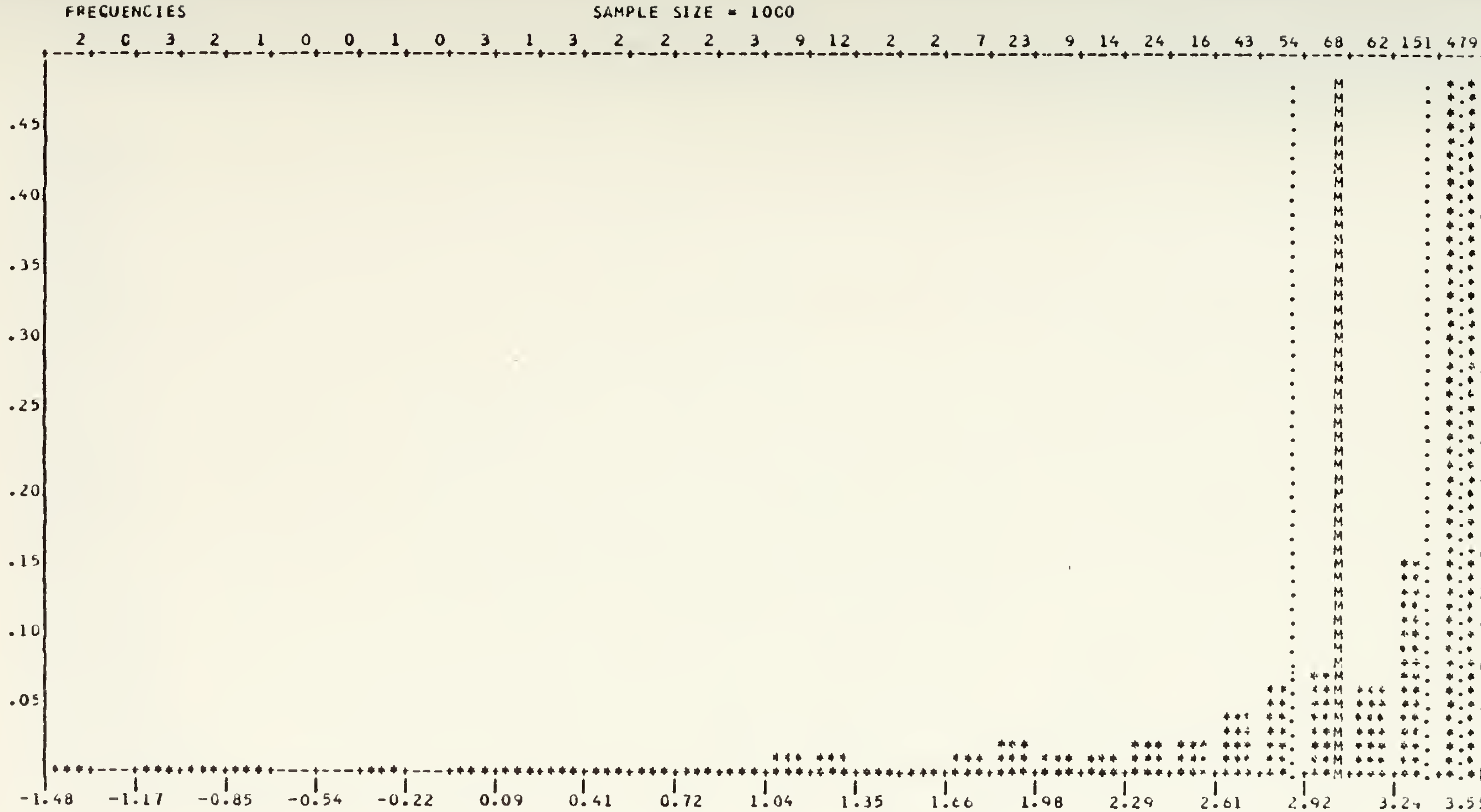
DISTRIBUTION	
MINIMUM	-2.276163E+00
.10 QUANTILE	-8.812172E-01
.25 QUANTILE (HINGE)	-2.518084E-01
.50 QUANTILE (MEDIAN)	4.928679E-01
.75 QUANTILE (HINGE)	1.293608E+00
.90 QUANTILE	2.072449E+00
MAXIMUM	4.666382E+00

16. Log transform of Long-tailed log-normal h down times.



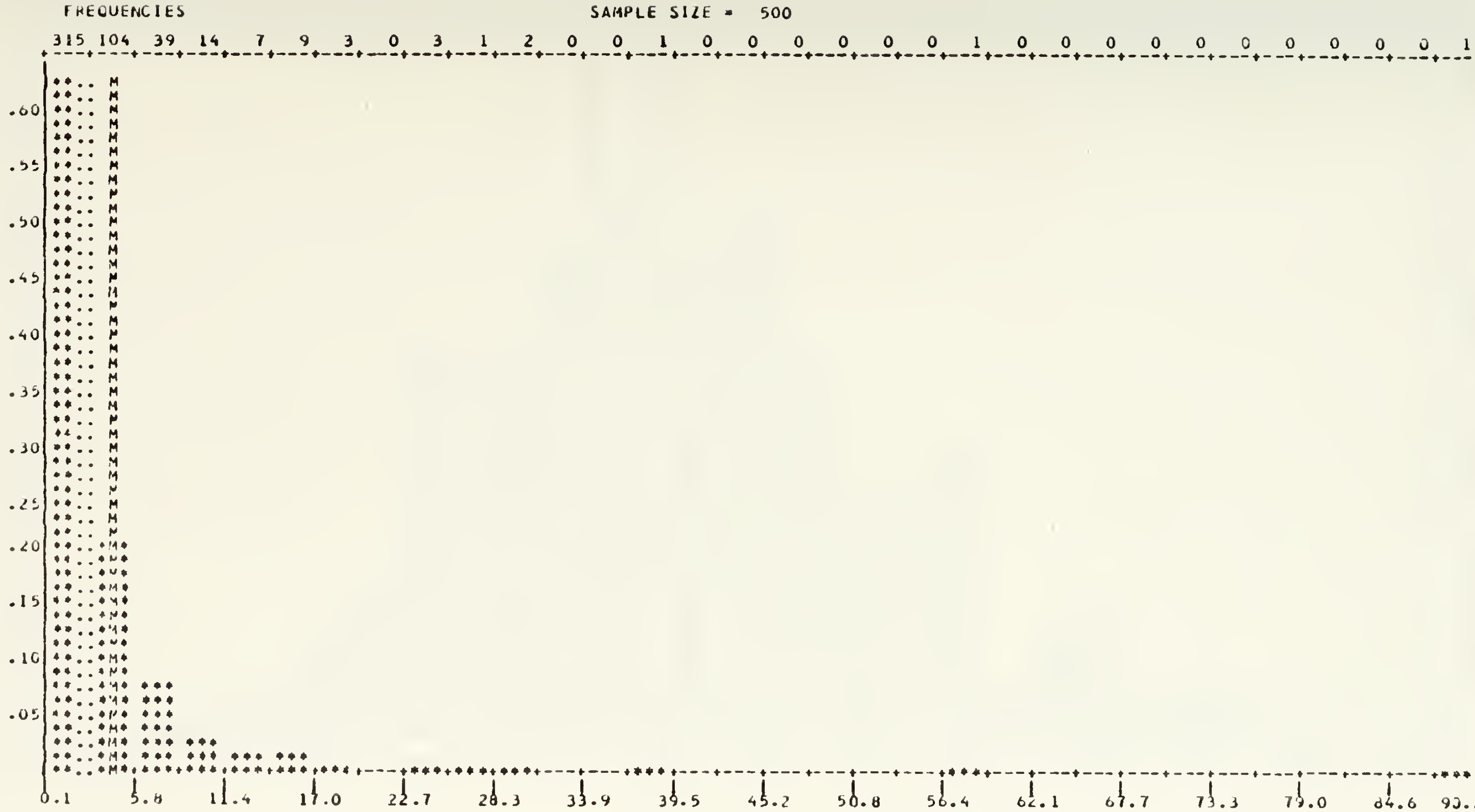
CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION		
MEAN	3.013350E+00	VARIANCE	1.226018E+00	M3	-1.340359E+01	MINIMUM		-1.666919E+01
MEDIAN	3.187012E+00	STD DEV	1.107257E+00	M4	2.004833E+02	.10 QUANTILE		2.983393E+00
TRIMEAN	3.187317E+00	COEF VAR	3.674504E-01	SKEWNESS	-9.873621E+00	.25 QUANTILE	(HINGE)	3.046875E+00
MIDMEAN	3.185122E+00	MEAN DEV	3.129861E-01	KURTOSIS	1.303782E+02	.50 QUANTILE	(MEDIAN)	3.187012E+00
MIDRANGE	-6.619385E+00	RANGE	2.009961E+01	BETA1	-1.336341E+01	.75 QUANTILE	(HINGE)	3.328369E+00
		MIDSPREAD	2.814941E-01	BETA2	1.996904E+02	.90 QUANTILE		3.392334E+00
						MAXIMUM		3.430420E+00

17. Pseudovalues of JK procedure for Exponential up and long-tailed log-normal down times.



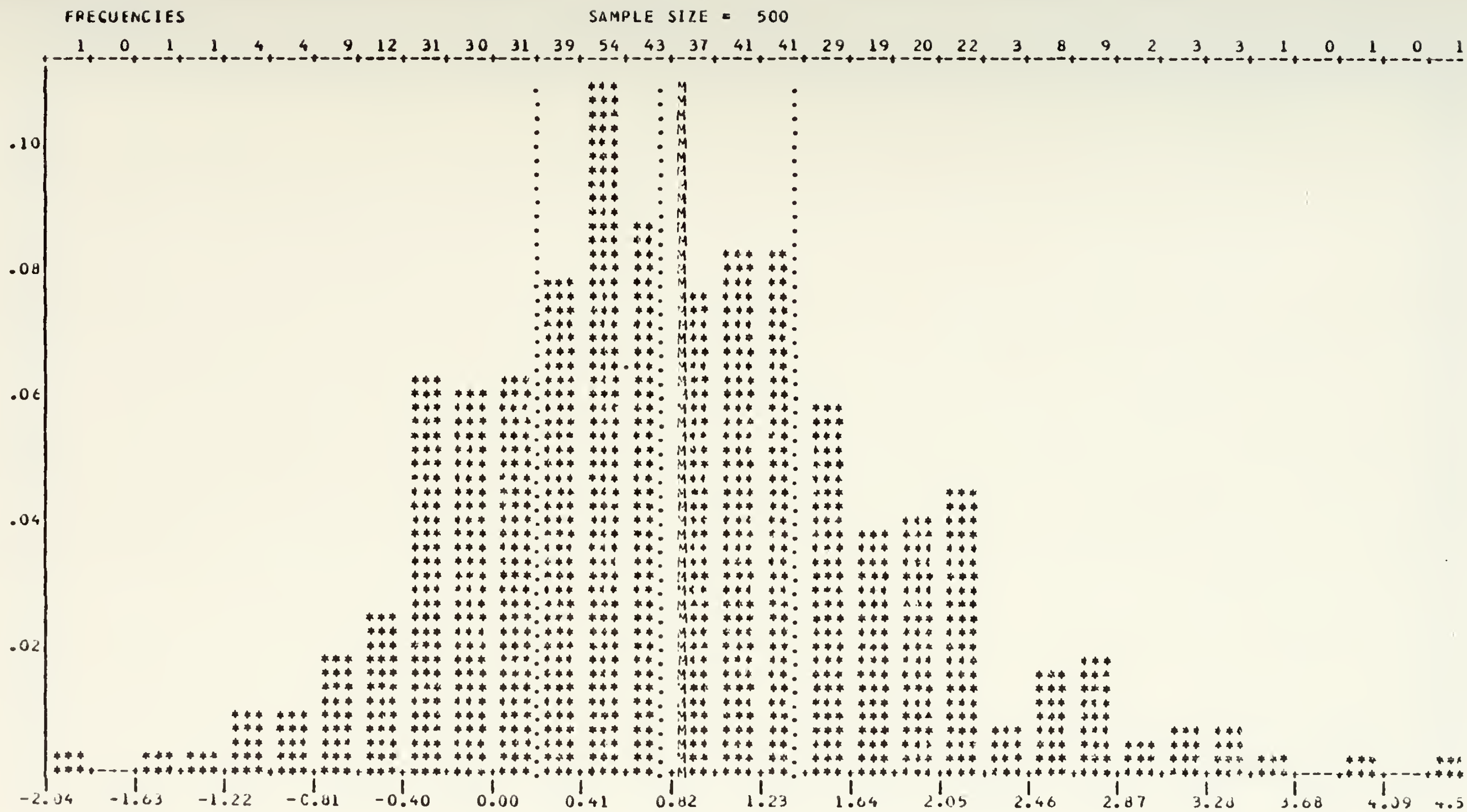
CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION	
MEAN	3.082485E+00	VARIANCE	5.296856E-01	M3	-1.115638E+00	MINIMUM	-1.482910E+00
MEDIAN	3.387451E+00	STD DEV	7.277951E-01	M4	3.755884E+00	.10 QUANTILE	2.277588E+00
TRIMEAN	3.305664E+00	COEF VAR	2.361063E-01	SKEWNESS	-2.893982E+00	.25 QUANTILE (HINGE)	2.932127E+00
MIDMEAN	3.334984E+00	MEAN DEV	4.125500E-01	KURTOSIS	1.038677E+01	.50 QUANTILE (MEDIAN)	3.387451E+00
MIDRANGE	1.035156E+00	RANGE	5.036133E+00	BETA1	-1.112294E+00	.75 QUANTILE (HINGE)	3.515625E+00
		MIDSPREAD	5.834961E-01	BETA2	3.742541E+00	.90 QUANTILE	3.528564E+00
						MAXIMUM	3.553223E+00

18. Pseudovalues for LNLJ procedure for Exponential up and long-tailed log normal down times.



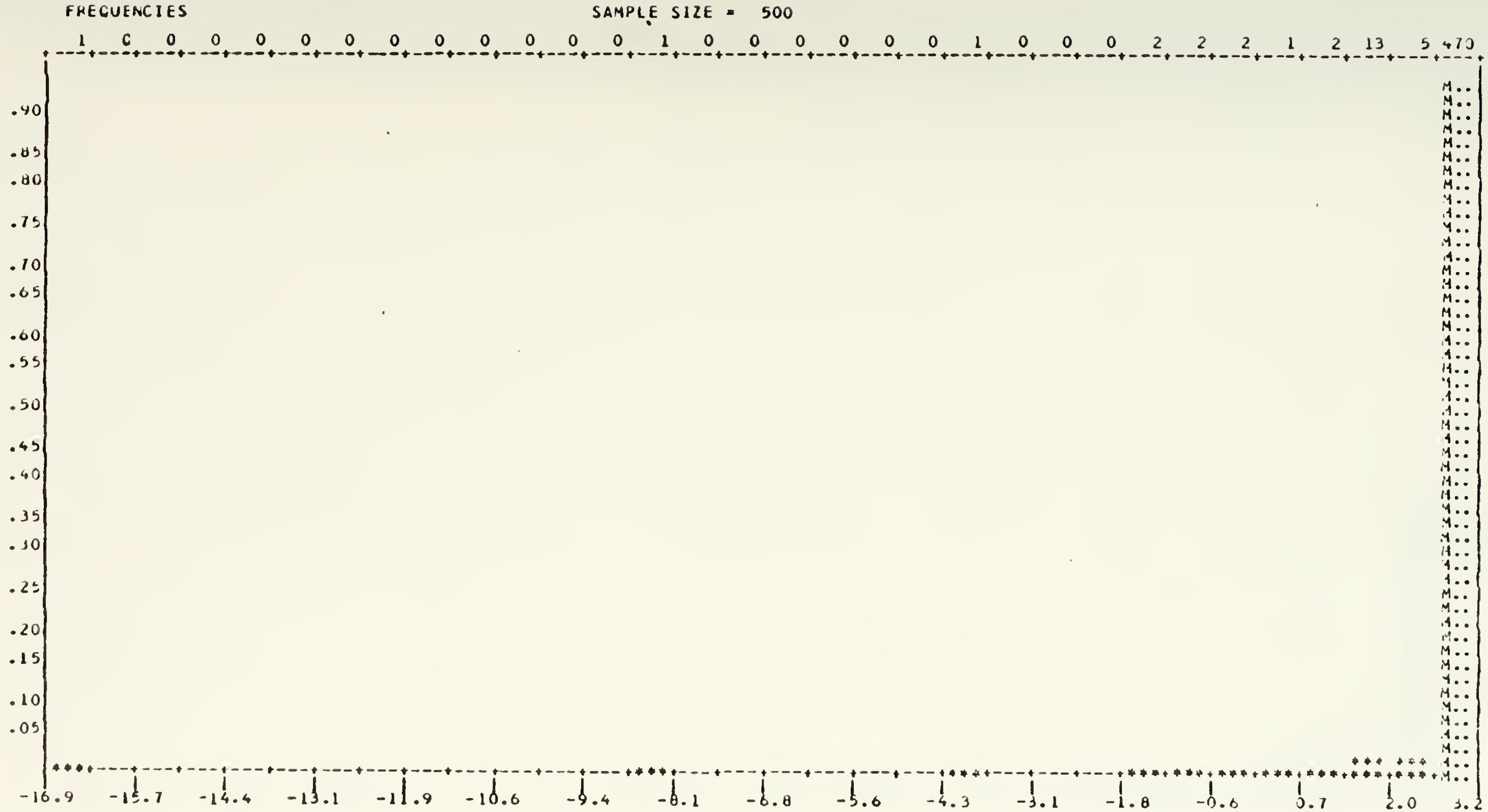
CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION	
MEAN	3.877219E+00	VARIANCE	3.991408E+01	M3	1.885141E+03	MINIMUM	1.300833E-01
MEDIAN	2.159245E+00	STD DEV	6.317759E+00	M4	1.351181E+05	.10 QUANTILE	7.474022E-01
TRIMEAN	2.369720E+00	COEF VAR	1.629456E+00	SKEWNESS	7.475751E+00	.25 QUANTILE (HINGE)	1.236163E+00
MIDMEAN	2.341661E+00	MEAN DEV	2.633169E+00	KURTOSIS	8.181276E+01	.50 QUANTILE (MEDIAN)	2.159245E+00
MIDRANGE	4.517908E+01	RANGE	9.009799E+01	BETA1	1.873846E+03	.75 QUANTILE (HINGE)	3.924224E+00
CEC4 MEAN	2.340988E+00	MIDSPREAD	2.688061E+00	BETA2	1.340562E+05	.90 QUANTILE	7.903708E+00
HARM MEAN	1.584791E+00					MAXIMUM	9.022809E+01

19. Long-tailed log-normal down times after grouping.



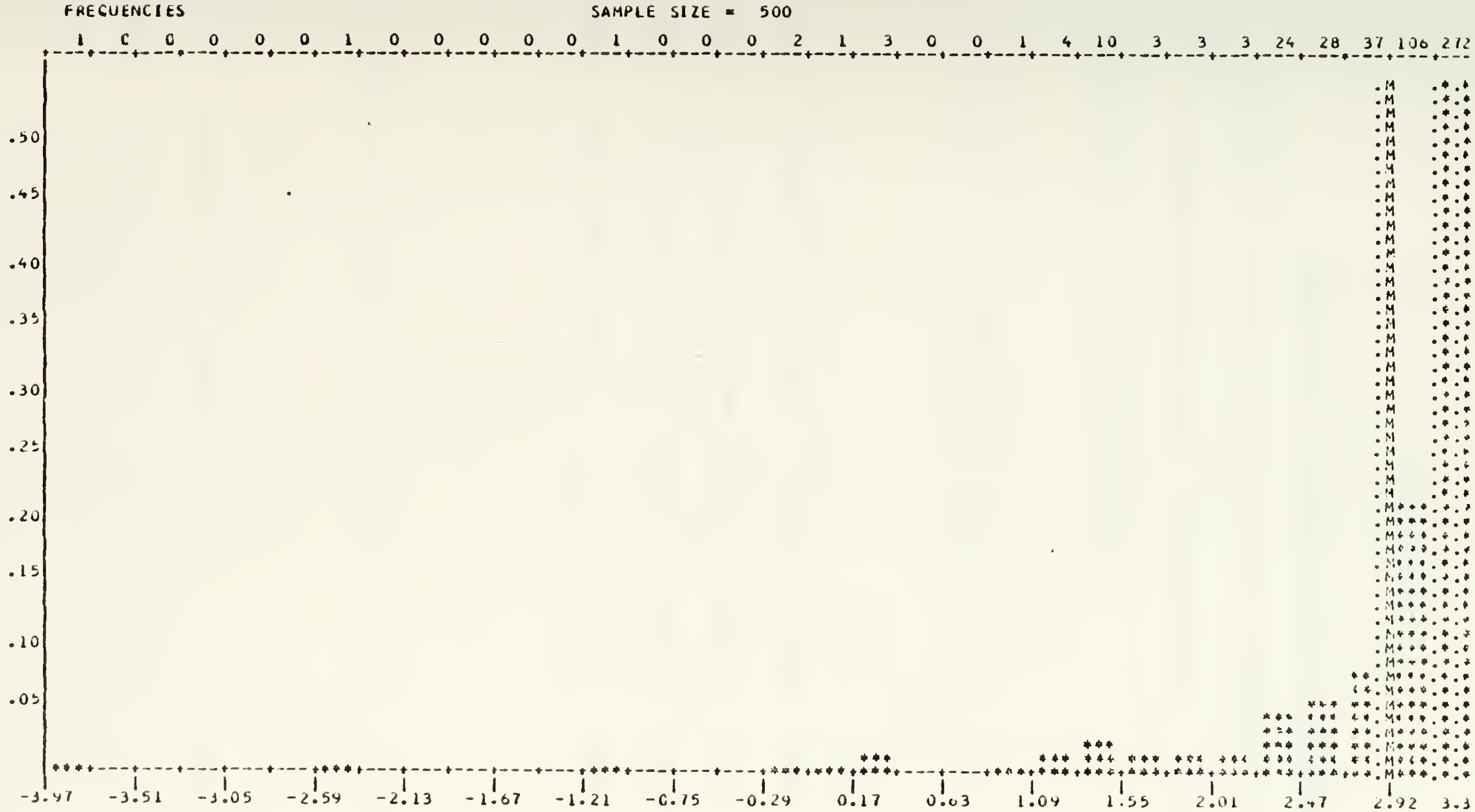
CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION	
MEAN	8.505730E-01	VARIANCE	8.668774E-01	M3	3.440958E-01	MINIMUM	-2.039579E+00
MEDIAN	7.657582E-01	STD DEV	9.310625E-01	M4	2.631876E+00	.10 QUANTILE	-2.911518E-01
TRIMEAN	7.796738E-01	COEF VAR	1.094629E+00	SKEWNESS	4.263270E-01	.25 QUANTILE (HINGE)	2.120127E-01
MIDMEAN	7.952968E-01	MEAN DEV	7.316915E-01	KURTOSIS	5.022736E-01	.50 QUANTILE (MEDIAN)	7.697582E-01
MIDRANGE	1.231380E+00	RANGE	6.541920E+00	BETA1	3.420342E-01	.75 QUANTILE (HINGE)	1.367167E+00
		MIDSPREAD	1.155153E+00	BETA2	2.619812E+00	.90 QUANTILE	2.067331E+00
						MAXIMUM	4.502340E+00

20. Log-transform of long-tailed log-normal down times after grouping.



CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION	
MEAN	2.895419E+00	VARIANCE	1.426158E+00	M3	-1.998091E+01	MINIMUM	-1.691406E+01
MEDIAN	3.112061E+00	STD DEV	1.194219E+00	M4	3.504402E+02	.10 QUANTILE	2.836182E+00
TRIMEAN	3.091919E+00	COEF VAR	4.124510E-01	SKEWNESS	-1.173178E+01	.25 QUANTILE (HINGE)	2.979492E+00
MIDMEAN	3.100163E+00	MEAN DEV	2.656836E-01	KURTOSIS	1.692974E+02	.50 QUANTILE (MEDIAN)	3.112061E+00
MIDRANGE	-6.846802E+00	RANGE	2.013452E+01	BETA1	-1.986119E+01	.75 QUANTILE (HINGE)	3.164063E+00
		MIDSPREAD	1.845703E-01	BETA2	3.476609E+02	.90 QUANTILE	3.176270E+00
						MAXIMUM	3.220459E+00

21. Pseudovalues of JK procedure for Exponential up and long-tailed log-normal down times after grouping.



CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION	
MEAN	2.969506E+00	VARIANCE	4.825097E-01	M3	-1.556422E+00	MINIMUM	-3.970215E+00
MEDIAN	3.197510E+00	STD DEV	6.946292E-01	M4	7.971553E+00	.10 QUANTILE	2.460391E+00
TRIMEAN	3.165314E+00	COEF VAR	2.339208E-01	SKEWNESS	-4.643745E+00	.25 QUANTILE (HINGE)	2.934814E+00
MIDMEAN	3.172955E+00	MEAN DEV	3.403525E-01	KURTOSIS	3.123976E+01	.50 QUANTILE (MEDIAN)	3.197510E+00
MIDRANGE	-2.928467E-01	RANGE	7.354736E+00	BETA1	-1.547096E+00	.75 QUANTILE (HINGE)	3.331421E+00
		MIDSPREAD	3.966064E-01	BETA2	7.910567E+00	.90 QUANTILE	3.363037E+00
						MAXIMUM	3.384521E+00

22. Pseudovalues of LNLJ procedure for Exponential up and long-tailed log-normal down times after grouping.

APPENDIX D

ONE-SIDED AVAILABILITY ESTIMATION

Best and worst cases for LNLJ procedure were simulated for one-sided availability estimation, and results are shown in Tables 17 and 18 as follows;

Table 17: Exponential Up and Log-normal Down Times

	N=15		N=25	
	JK	LN	JK	LN
Low. Availability Bound Cover	0.999	1.000	0.999	1.000
Upp. Availability Bound Cover	0.933	0.899	0.966	0.965
Avg. Lower Availability Bound	0.889	0.879	0.910	0.908
Avg. Upper Availability Bound	0.966	0.964	0.966	0.966

Table 18: Exponential Up and Weibull (k=1/2) Down Times

	N=15		N=25	
	JK	LN	JK	LN
Low. Availability Bound Cover	0.900	0.990	0.914	0.997
Upp. Availability Bound Cover	0.966	0.853	0.955	0.705
Avg. Lower Availability Bound	0.844	0.460	0.881	0.500
Avg. Upper Availability Bound	0.972	0.970	0.975	0.955

Notice the low Average Lower Availability Bound provided by the LNLJ procedure for the Weibull (k=1/2). Results show that the one-sided availability estimation behaves as the two-sided availability estimation.

LIST OF REFERENCES

1. Swedish Board for Technical Development STU-Report 75-4090, Test Plans for Availability, by J. L. Rise and O. Bjorklund, p. 1-3, December 1978.

2. Gaver, D. P., and Chu, B. B., "Jackknife Estimates of Component and System Availability," Technometrics, Vol.21, No. 4, p. 443-449, November 1979.

3. Klein, M. B., and Almog, R., Suitability of The Log-normal Distribution for Repair Times, paper presented at International Conference on Reliability and Maintainability, Lannion, France, 8-12 September 1980.

4. Miller, R. G., "The Jackknife - a Review," Biometrica, Vol. 61, No. 1, p. 1-15, 1974.

5. Mosteller, F., and Tukey, J. W., "Data Analysis, Including Statistics," Chapter in Handbook of Social Psycology, Vol. 2, Addison- Wesley , 1968- 1970.

6. Reeds, T., "On Jackknifing Maximum Likelihood Estimator," Annals of Statistics, No. 6, p. 727-739, 1978

7. Mosteller, F., and Tukey, J. W., Data Analysis and Regression, p. 205, 358-364, Addison- Wesley , 1977.

8. Dixon, W. J., and Massey, F. J. Jr., Introduction to Statistical Analysis, p. 330-332, McGraw-Hill , 1969.

9. Johnson, N. L., and Kotz S., Continuous Univariate Distributions, Vol. 1, p. 115, John Wiley and Sons Inc., 1970.

INITIAL DISTRIBUTION LIST

No. Copies

1. Defense Technical Information Center 2
Cameron Station
Alexandria, Virginia 22314

2. Library, Code 0142 2
Naval Postgraduate School
Monterey, California 93940

3. Department Chairman, Code 55 1
Department of Operations Research
Naval Postgraduate School
Monterey, California 93940

4. Professor D. P. Gaver, Code 55GV 4
Department of Operations Research
Naval Postgraduate School
Monterey, California 93940

5. Bogazici Universitesi 1
Yon Eylem Analizi Bolumu
Besiktas, Istanbul, Turkey

6. Dz.K.K.ligi 2
Bakanliklar, Ankara, Turkey

7. Deniz Harpokulu K.ligi 1
Heybeliada, Istanbul, Turkey

8. Ortadogu Teknik Universitesi 1
Yon Eylem Analizi Bolumu
Ankara, Turkey

9. Ltcdr. K. N. Flaathen 1
Neptunv. 6
5071 Loddefjord, Norway

10. Cpt. J. N. Lott 1
102 Malloway Lane
Monterey, California 93940

11. Lt. L. Yavuz 1
Baglarbasi Mah.
Cengiz Topel Cad.
5. Sok No.1
Kirikkale, Turkey

12. Lt. Davut Kirca 1
Ipras Sitesi 7. Sok No.4
Tutunciftlik, Kocaeli, Turkey

13. Professor R. N. Forrest, Code 55FO 1
Department of Operations Research
Naval Postgraduate School
Monterey, California 93940

14. LT Barbaros Aba 1
Denizalti Filosu K.ligi
III Filotilla Komodorlugu
Golcuk-Kocaeli-TURKEY

Thesis
A1135
c.1

195553

Aba

Investigation of alternative methods including jackknifing for estimating point availability of a system.